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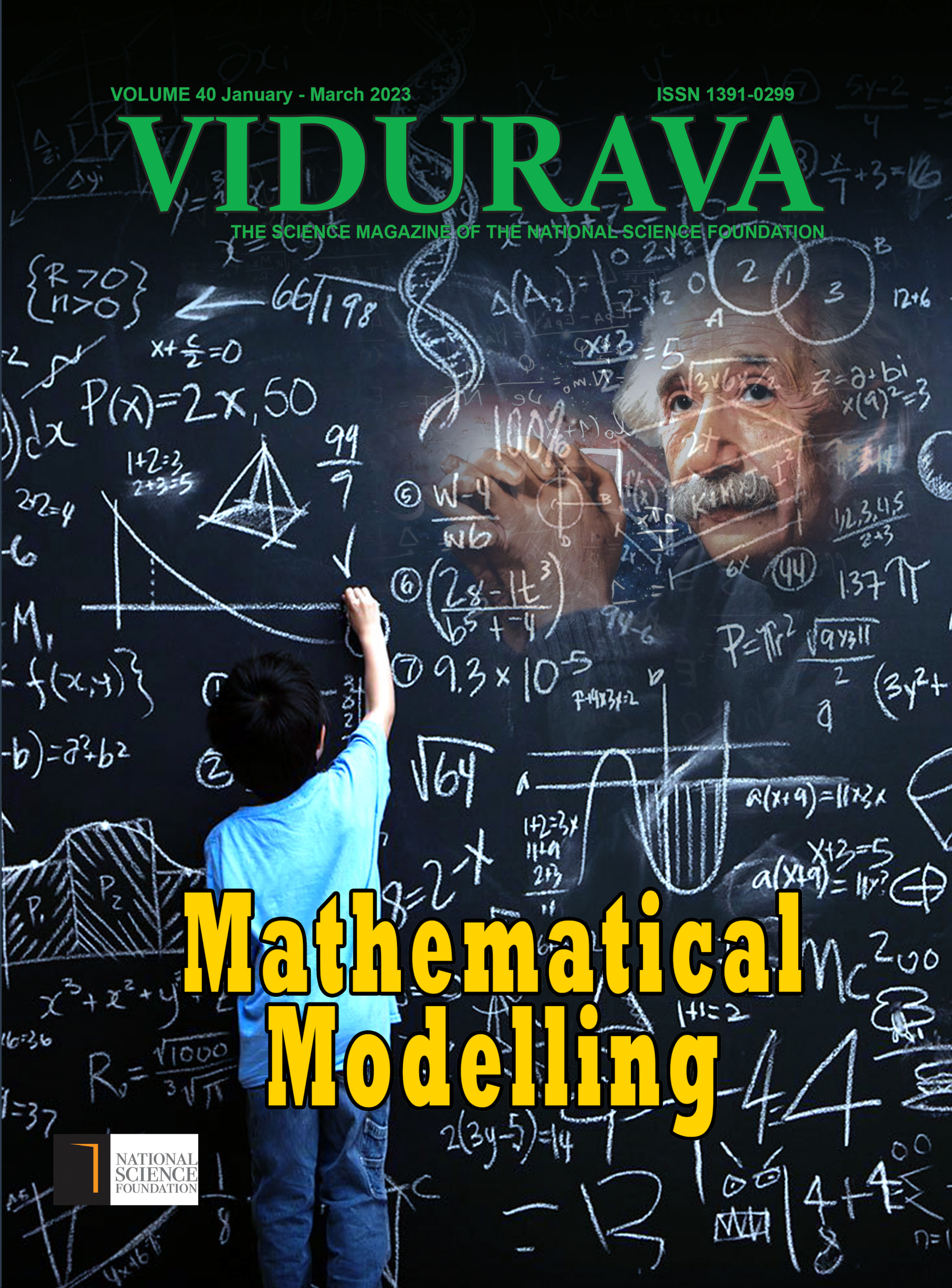

Mathematical Modelling

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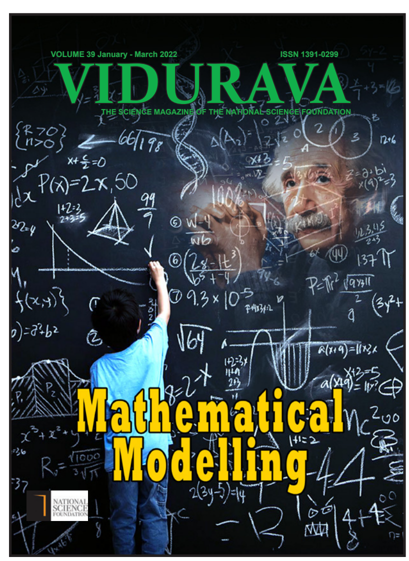

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Editorial

Mathematical Modelling: A Modem for Manipulation and Innovation

The theme of this edition of *Vidurava* is on Mathematical Modelling, which provides a pathway for creativity. Nevertheless, some at least of the material information in these articles require an advanced knowledge of mathematics, and hence may sometimes be beyond the understanding of the common readership of this magazine.

Contextually however, these articles provide extremely valuable information on,

1) mechanisms for quantifying disaster risks, and assessing the consequences of natural disasters,

2) reviewing the evolutionary process of global languages, and the unfortunate disappearance of native languages of aboriginals and other ancient populations despite protection guaranteed by the United Nations Organization

3) the somewhat confusing manipulations of what is referred to as 'Real Life Fussy Modelling',

4) computing and constructing models for environmental pollution,

5) developing mathematical models for assessing and computing warranty periods for manufactured utility goods, equipment, instruments and machinery, and

6) evolving diagnostic procedures for outbreaks of virulent infectious diseases such as the Corona virus disease, and assessing the spread and mortality rates resulting from such diseases.

It is hoped that the information generated by the articles in this magazine would enlighten the readers of the utility value of mathematical modelling.

M. Asoka T. De Silva

Mathematical models (modelling)

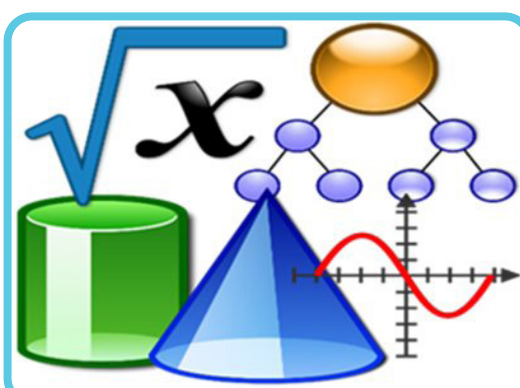
Prof. Sanjeewa Perera

What is a (Mathematical) model?

I am sure all of us have experiences in making *sand pittu* in our childhood. Just go back to the memories of your childhood and what you had done when you were at the beach (Figure 1). I am sure, all of us would have prepared sand castles. Since sand is used to demonstrate real objects, the resultant outcomes are considered as models.

Mathematical modelling

Mathematical modelling is the art of translating real world problems in to mathematical terms, usually in the form of equations to make predictions or provide insight. Models describe our beliefs about how the particular phenomena functions. In mathematical modelling, those beliefs of real systems are translated into the mathematics world using mathematical concepts, symbols, operators *via* the language of mathematics. A mathematical world consists of different mathematical objects such as a system of equations, a set of ordinary differential equations, a set of partial equations, a set of stochastic equations, geometric or algebraic structures, algorithms or even very simple objects like sets of numbers and other symbols. In our school mathematics, we work with simultaneous equations (Figure 2). For example, if the price of a pen and the price of a book are given, we can find how many pens and books we can buy. To find solutions, models (simultaneous equations) were developed. What does this mean? Even in school we use mathematical modelling concepts or mathematical models to solve real world problems.



Thus, modelling is a cognitive activity in which we

What is Mathematics

Mathematics is the science that deals with the logic of shape, quantity, and arrangement. Mathematics is all around us, in everything we do. It is the building block for everything in our daily lives, including mobile devices, computers, software, architecture (ancient and modern), art, money, engineering and even sports. However, we never see mathematics behind our daily life because most of mathematical activities consist of discovering and proving properties of abstract objects. Since image is not clear, most of us think “Mathematics mean staying and understanding abstract objects”. Mathematics can be used as a tool to understand / explain / describe / predict real life phenomena. This is what scientists and engineers do, they use mathematics as a pathway or tool to find solutions to real life problems. This modern mathematical tool is known as “Mathematical Model” and process is known as “Mathematical Modelling”.

Figure 1 : Models; Experiences at the early childhood

think about and make models to describe how objects of interest behave. Commencing from our childhood memories of casting sand castles on beaches to peering into the heart of the atom, and even to understand the future climate, we need modelling. With modelling we can travel to the edge of the universe, allowing us to design the correct technology of the future.

There are many ways in which we can describe the behaviour of objects. For example, by drawing sketches (house plans), physical models (building prototype house models before real construction begins), computer programme or using mathematical formulae.

Mathematical modelling steps

Principles of mathematical modelling and the steps to be taken are summarized by raising very simple questions, and answering such questions.

1. **Why?** What are we looking for? Identify the need for the model.
2. **Find?** What do we want to know? List the data we are seeking.
3. **Given?** What do we know? Identify the available relevant data.
4. **Assume?** What can we assume? Identify the circumstances that apply.
5. **How?** How should we look at this model? Identify the governing physical principles.
6. **Predict?** What will our model predict? Identify the equations that will be used, the calculations that will be made, and the answers that will result.
7. **Valid?** Are the predictions valid? Identify tests that can be made to validate the model, i.e., is it consistent with its principles and assumptions?
8. **Verified?** Are the predictions good? Identify tests that can be made to verify the model, i.e., is it useful in terms of the initial reason it was done?
9. **Improve?** Can we improve the model? Identify parameter values that are not adequately known, variables that should have been included, and/or assumptions/ restrictions that could be lifted. Implement the iterative loop that we can call "model-validate-verify-improve predict."
10. **Use?** How will we exercise the model? What will we do with the model?

This list of questions and instructions is not an algorithm for building a mathematical model. However, it indicates ideas that are essential for mathematical modelling, as they are the key steps for problem formulation in general. Thus, we should expect the individual questions to recur often during the modelling process, and we should regard this list

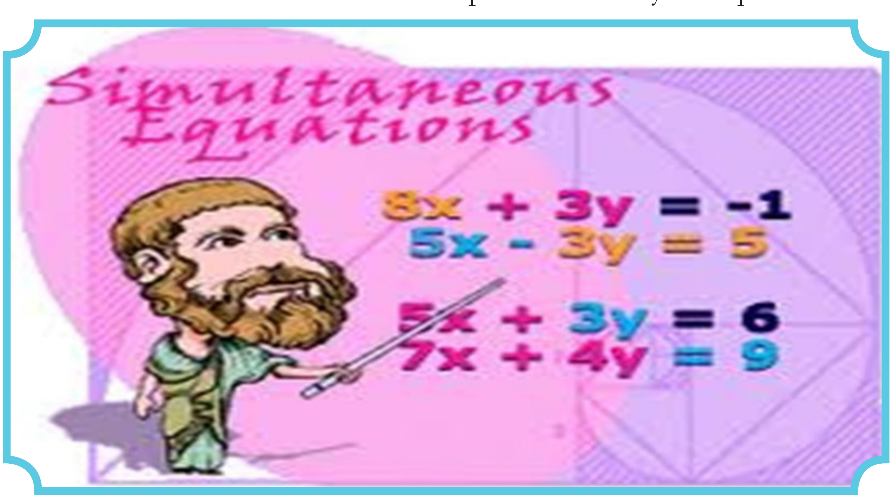
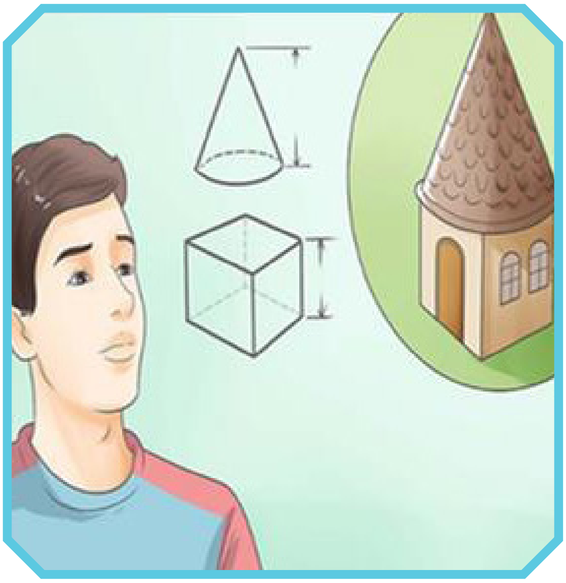


Figure 2 : Mathematical Model; Experiences from school



Mathematical Models: Mathematical Objects are used
Therefore, a model can be defined as a miniature representation of something, a pattern of something to be made, an example for imitation, or a description to visualize something that cannot be directly observed etc.. If mathematical objects are used to demonstrate real phenomena, the resulting outputs can be referred to as Mathematical Models.

as a general approach to ways of thinking about mathematical modelling.

Imagine, we want to estimate how much power could be generated by a wind power plant to be established at some location. Before setting up the power plant, it is worth to develop a model and do comprehensive analysis of the entire situation, and then identify the most feasible plan. It is very clear that this kind of analysis cannot be done experimentally, since it is too costly as well as not safe. We could try to answer said 10 questions throughout the model development stage.

Historical mathematical models

Lotka-Volterra model

A predator is an organism that eats another organism. The prey is the organism which the predator eats. Some examples of predator and prey are lion and zebra, bear and fish, and fox and rabbit (Canadian lynx and snowshoe rabbit). This model is known as Lotka-Volterra model and was developed in 1900 by considering the rate of change of two populations (Figure 3).

Hodgkin–Huxley model

The Hodgkin–Huxley model, or conductance-based model, is the first mathematical model that

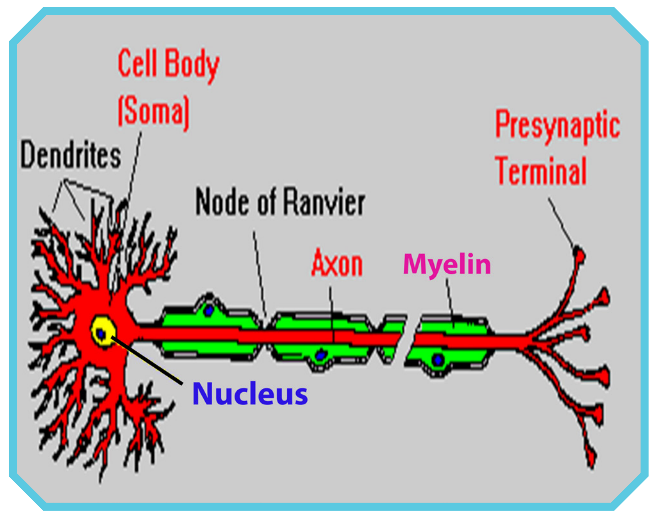


Figure 4 : Electrophysiology of the cell

describes how action potentials in neurons are initiated and propagated. In 1963 Hodgkin and Huxley received the Nobel Prize in Physiology or Medicine for this work (Figure 4).

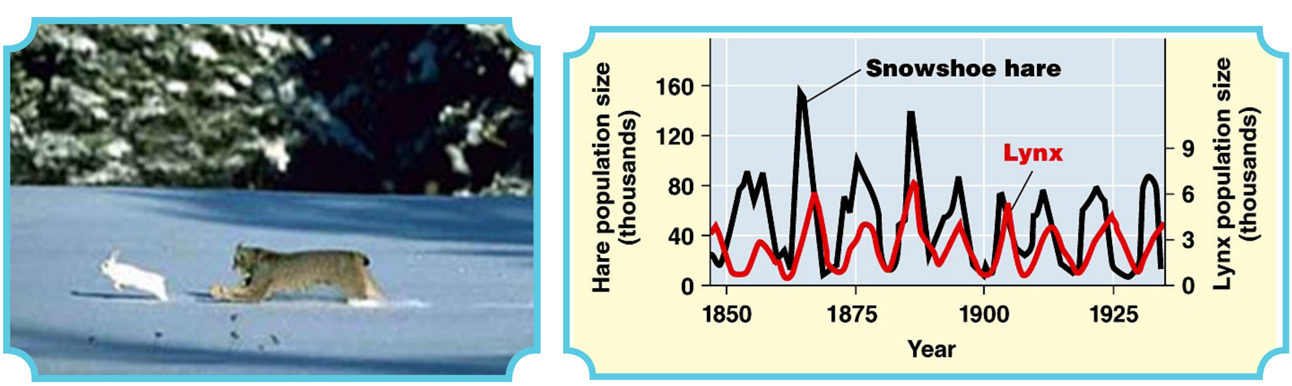


Figure 3 : Canadian lynx *vs* snowshoe rabbit: Population dynamics

Tumor and cancer models

Mathematical models have been developed that describe tumor and cancer progression, which help to predict response to therapy.

Immunology models

Immunology mathematical models have also been developed to answer the following questions.

❖ How do immune cells find a bacterial target?
❖ Under what conditions can the immune system control a localized bacterial infection?
❖ If the immune system fails, how will the bacteria spread in the tissue?

Models for day-to-day

For some of the day-to-day activities (personal or industrial), mathematical modelling concepts are used.

Antibiotics can be life saving for critically ill patients. However, overuse or unnecessary administration can cause antibiotic resistance, which has led to major difficulties in the management of infected patients. By using mathematical models, we can reproduce the empirical patterns observed, thereby to understand how antibiotic usage patterns may be optimized. As an example, for some patients, antibiotics are administered at 12 hour intervals, which for some it is just 8 hour intervals. Thus, mathematical models are an important step in regulating the consumption of antibiotics.

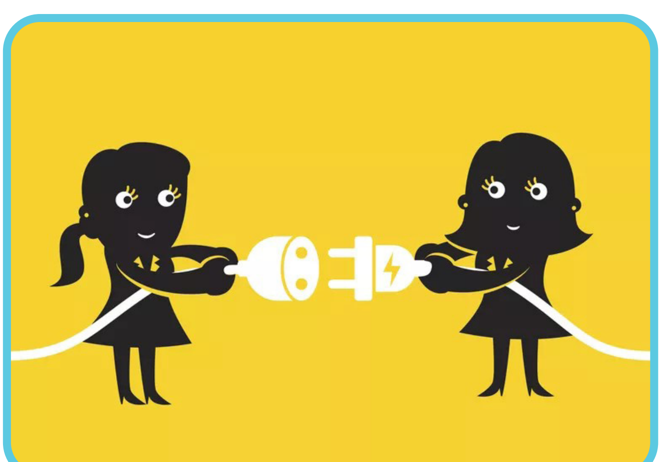
Surrounded by the Indian Ocean, fish are an important source of food in Sri Lanka. However, overfishing may lead to a worrying situation with the extinction of many species. A particular interesting element of marine ecosystems is that large fish eat small fish. Overfishing large species such as tuna and seer fish may lead to an increase in the abundance of smaller species on which big fish feed. This increase of smaller fish may lead to a decrease of species on which smaller fish feed. Thus, the situation may become catastrophic. How do we stabilize it? Coupling mathematical modelling with ecological dynamics provides a better understanding of the dynamics of fisheries systems.

In designing new aircrafts, prior to the design, the objectives and the specifications of the aircraft are done. Usually these are done using drawings and mathematical equations. Then an aerodynamic description of the aero plane is conducted. Aerodynamic modelling is concerned with the development of mathematical models to describe the forces and moments acting on the airframe. Then comes the simulation stage, where computers are used to do the initial simulations of the aircraft. Afterwards, small models of the design or just certain parts of the plane are made and tested in wind tunnels to test and experience its aerodynamics. Thus, mathematical modelling plays a major role in designing aircrafts.

The examples discussed above, are in the fields of Biology, Medicine, Physics, Engineering and Industrial (manufacturing) process. Mathematical Biology, Mathematical Physics, Engineering Mathematics and Industrial Mathematics are some known subjects which describe mathematical relationship in such fields.

Mathematical Finance, also known as Quantitative Finance and Financial Mathematics, is a subject, concerned with mathematical modelling of financial markets.

Insurance is a social mechanism that allows individuals and organizations to compensate economic losses caused by



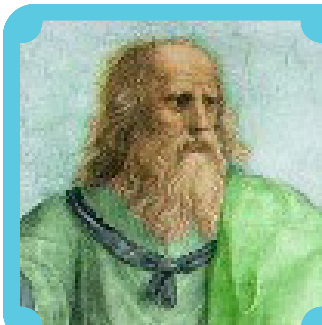
Models in Economics
Supply and demand models, in economics, relationship between the quantity of a commodity that producers wish to sell at various prices and the quantity that consumers wish to buy. Mathematical Economics or Quantitative Economics is the subject which describe mathematical relationship in Economics.

Magnificent Mathematics Quotes

"The uniform character of mathematics is the essence of science, for mathematics is the foundation of all exact scientific knowledge."

David Hilbert , 1862 – 1943

Mathematician



"Mathematics is the language in which the gods speak to people"

Plato, C. 427 BC – C. 347 BC

Mathematician and Philosopher

"Simple laws can very well describe complex structures. The miracle is not the complexity of our world, but the simplicity of the equations describing that complexity."

Sander Bais, 1945

Theoretical Physicist

unfavourable events. Actuarial Mathematics is the mathematical theory which deal with insurance. Not only that, Environment Modelling, Ecological Modelling, Disaster Modelling, Social Modelling, Marketing Modelling and even Mathematical Models to describe "how to spread gossip" use mathematical modelling. Some of the modelling examples are demonstrated in the next six articles. These include disaster risk, language dynamics, fuzzy uncertainty, air pollution, warranty period and infectious disease transmission models.

Mathematical Modelling can be considered as a multidisciplinary actions tool to handle real problems. In conclusion, the mathematical modelling process provides mathematicians to be Ecologists, Biologists, Economists etc..

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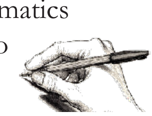
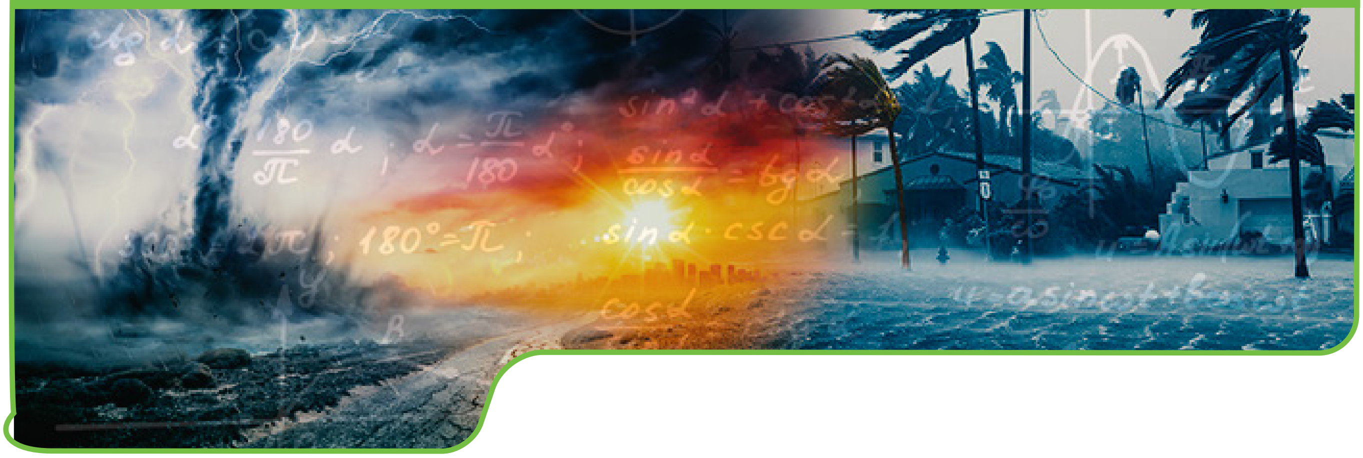


Mathematics for quantifying disaster risk

Dr Naleen Ganegoda



Natural disasters such as floods, landslides, cyclones, earthquakes, droughts, volcanoes and tsunamis affect millions of people worldwide every year. There is no exception for Sri Lanka too. Many of the disasters occur as a result of adverse weather conditions. Flooding is a disaster due to extremely high rainfall, while drought and wildfires occur at the other extreme, where shortage of rainfall occurs for longer periods. The most devastating disaster experienced by Sri Lanka in the recent history was the tsunami in 2004. Minimizing damage is the only option available, as no one can stop natural disasters. Disaster management can be identified as the process in minimizing damages subject to available facilities.
In the long run, designing risk indices, streamlining early warning systems, analysing past disasters, and deciding mitigation efforts are the key aspects to be considered. In this regard, risk indicators play an important role. In this article, mathematical formulations of several indicators, and their utility value in disaster management are discussed.

What is a risk indicator?

In the context of a natural disaster, a risk indicator measures the likelihood of occurring damage, often using mathematical formulae or models. Indicators are used to predict the potential risk ahead rather than to indicate the damage caused by a past disaster. Designing a risk indicator is a challenging task, as disasters are related to many aspects of social, geographical, economic and demographical issues. Some indicators may cover the overall aspect, as for example, the number of people affected by floods. Furthermore, for comparative purposes, one may estimate the number of people affected per one million of the total population.

Why do we need indicators?

The main utility value of a risk indicator is to assess the potential damage. This enables the relevant authorities responsible for rescue operations to plan necessary actions to mitigate damages. Most importantly, warning systems can be established since prevention is always better than cure. In addition, administrators can take policy decisions in respect of structuring insurance schemes, as well as paying compensation for damages. For instance, the government can arrange to distribute financial allocations for flood relief, based on a risk indicator of affected land areas in each divisional secretariat. In the long-run, such indicators are important to oversee different phases of disaster management such as prevention, mitigation, preparedness, response and recovery.

Risk indicators

When a country faces a natural disaster, it is important to estimate its economic and social impacts. Here, four such indicators (Formula 1, 3, 4 and 5) are presented, which were tested in a programme for Latin America and the Caribbean countries as reported by O.D. Cardona in a summary report (2008): Indicators of Disaster Risk and Risk Management. Our aim is to understand how basic mathematics can be incorporated in designing risk indicators. The first indicator is the Local Disaster Index (LDI), that is designed to

$$\text{Local Disaster Index (LDI)} = LDI_{Deaths} + LDI_{Affected} + LDI_{Losses} \text{ --------- (1)}$$

Formula 1

$$\text{Local Disaster Index (LDI)} = 0.5\ LDI_{Deaths} + 0.25\ LDI_{Affected} + 0.25\ LDI_{Losses} \text{ --------- (2)}$$

$$\text{Disaster Deficit Index (DDI)} = \frac{\text{Maximum Considered Event Loss (MCE Loss)}}{\text{Economic Resilience (ER)}} \text{ --------- (3)}$$

Formula 2 and Formula 3

$$\text{Prevalent Vulnerability Index (PVI)} = \frac{PVI_{Exposure} + PVI_{Fragility} + PVI_{Lack\ of\ Resilience}}{3} \text{ --------- (4)}$$

Formula 4

$$\text{Risk Management Index (RMI)} = \frac{RMI_{RI} + RMI_{RR} + RMI_{DM} + RMI_{FP}}{4} \text{ --------- (5)}$$

Formula 5

indicate the risk of low-hazard recurrent disasters in a certain locality. Flooding that occurs annually across a country is an example for such a disaster. In the Sri Lankan context, this index can be calculated for divisional secretariat areas, to identify divisions that need more attention. The Local Disaster Index (LDI) is associated with a cumulative impact of deaths, the affected number and losses (Formula 1).

Each sub-index can be basically calculated as the number of persons dead, affected and the economically lost viable assets such as household, crops, jobs etc.. For uniformity in comparisons, each sub-index should be normalized by dividing the land area of the locality. Such normalizing is required to supress overestimations due to high populations in larger areas. This formula can be modified with different weights on sub-indices. For instance one can assign the weights 0.5, 0.25 and 0.25 adding into 1 as follows to indicate twice of risk on deaths compared to other cases (Formula 2).

The next indicator is the Disaster Deficit Index (DDI)(Formula 3). Disaster Deficit Index (DDI) shows the macroeconomic impact of a disaster in relation to financial ability to deal with that disaster. Once a risk indicator is designed as a fraction, numerator should be placed with directly proportional measures, while inversely proportional measures are set in the denominator. Maximum Considered Event Loss (MCE Loss) is a probabilistic measure that estimates the potential impact of an extreme disaster for a return period. Economic Resilience (ER) enumerates internal and external resources to cope with the disaster at the time of evaluation. One can use DDI to compare the ability of facing a disaster by different countries. Furthermore, a threshold can be defined to distinguish good and bad performers. Higher DDI shows higher risk as the gap between the loss and resilience is higher. For instance, countries having DDI greater than 1 are considered to be bad performers. According to the above programme, Bolivia has the highest DDI of 5.7 and Costa Rica has the lowest DDI of 0.76, where Costa Rica is the only country with DDI less than 1. DDI indicates future potential of managing a disaster measured *via* statistical inference rather than based on the impact of past events.

Table 1: Scale for RMI sub-indices

Linguistic description	Scale value
Low	1
Incipient	2
Significant	3
Outstanding	4
Optimal	5

Some indicators are important in their usual average form. For instance, the indicators in Formula 4 and 5 show the overall risk, assuming contributions from each sub-index in equal weight.

$PVI_{Exposure}$ stands for the susceptibility of population, and $PVI_{Fragility}$ is aligned with socio-economic factors such as poverty. An inverse relationship to human development is expected in $PVI_{\text{Lack of Resilience.}}$ Here, the three sub-indices should be consistent with the same measuring units.

$$\text{Risk} = \text{Hazard} \times \text{Exposure} \times \text{Vulnerability} \text{ -------- } (6)$$

Formula 6

$$R = H \times V \times E \text{ --------- } (7)$$

Formula 7

$$M_L = \log_{10} A - \log_{10} A_0 \text{ --------- } (8)$$

Formula 8

In Formula 5, the four sub-indices represent identification of risk, risk reduction, disaster management and governance, and financial protection, respectively. Quantification of these indices can be carried out in 1 - 5 scale. Thus, the final RMI is based on linguistic descriptions on each performance. Table 1 shows these descriptions.

An approach called fuzzy set theory can also be used that allows scale values for merging situations (eg. Scale value 1.5: better than low, but not that incipient).

The above indices LDI, PVI and RMI are in additive structure, where several sub-indices are added together for overall risk. A multiplicative structure can also be used as shown in formula 6. In an additive structure, we need all the sub-indices to be in the same units, where it is not necessary in a multiplicative case. If we consider the formula (1) of LDI, an increment of a sub-index yields the same increment as the overall index when the other sub-indices are fixed. However, this effect is different in a multiplicative case such as (6) (UNDRR Global Assessment Report, 2015), since the increment of a sub-index is multiplied by the product of other fixed sub-indices. These types of mathematical features should be incorporated properly for achieving realistic interpretations.

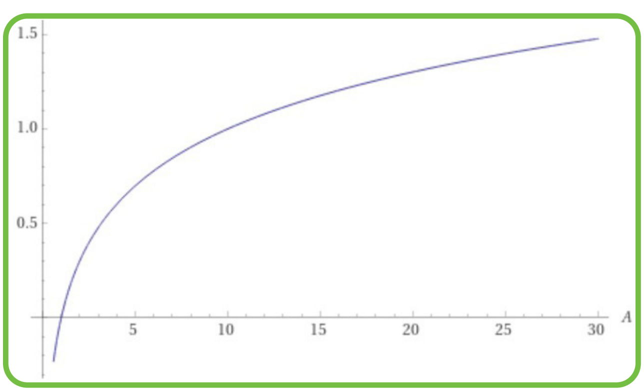


Figure 1: Logarithmic function $\log_{10}$A

According to the formula in (6), one can design an indicator for flood risk as shown in Formula 7. H stands for rainfall intensity or river-water levels, where the rise of each would create a more hazardous situation. The vulnerability sub-index V can be estimated as the number of threatened individuals who live in low-lying areas. It might include other characteristics that result from increased susceptibility of individuals. The exposure E should indicate the value of assets exposed to the disaster. The same formula has been used to indicate the risk of landslides with the sub-indices: H - probability of occurring a landslide of a given intensity, V - degree of loss expected from the landslide and E - value of the elements exposed to the landslide as used by S. Segoni and F. Caleca in their journal article in Land: Definition of Environmental Indicators for a Fast Estimation of Landslide Risk at National Scale. A similar approach can be implemented to determine a hurricane risk index.

Early warnings

Early warning systems can minimize the damages caused by a natural disaster. Such systems should respond effectively and efficiently learning from the early signs of a catastrophic phenomenon. For instance, people expect the warnings about a tsunami when an earthquake occurs beneath the ocean. Volcano eruptions and landslides near or beneath the ocean are also possible causes of a tsunami. In the case of an earthquake, a vertical displacement of the ocean floor leads the way to start tsunami waves that reach the coastal areas with a high amplitude. Here the magnitude of an earthquake given by the seismic monitoring, is the first quantification that triggers the warning. Thereafter, surface buoy sensors provide details of sea level changes. Proper maintenance of measuring and communication equipment is required for the determination of efficient and effective warning systems. Otherwise, people have to be alert on every earthquake that have underwater epicentres. Richter scale is the measurement used to identify the magnitude of an earthquake. Usually earthquakes with Richter scale values greater than 6, lead to structural damages and collapse of buildings that might occur if it is greater than 8. At present, a measurement called moment magnitude scale is used for more accurate estimates.

Several modified formulae are used for the Richter scale (M_L). Let us consider the following original form for observing its mathematical context (Formula 8).

Here, A represents the maximum amplitude (in mm) of the seismometer and A_0 is taken as a reference value, which is said to be the seismometer reading of a calibration earthquake. Thus, M_L has a non-linear relationship with A. Figure 1 illustrates the behaviour of $\log_{10} A$ against A. The behaviour of M_L can be taken by a downward vertical shift since $\log_{10} A_0$ is constant. Furthermore, according to the rules of logarithms, one can see (8) as $M_L = \log_{10} (A/A_0)$. Once we want to see A for a given M_L, the required formula is $A = A_0\, 10^{M_L}$ that incorporates an exponential function as the inverse of a logarithmic function.

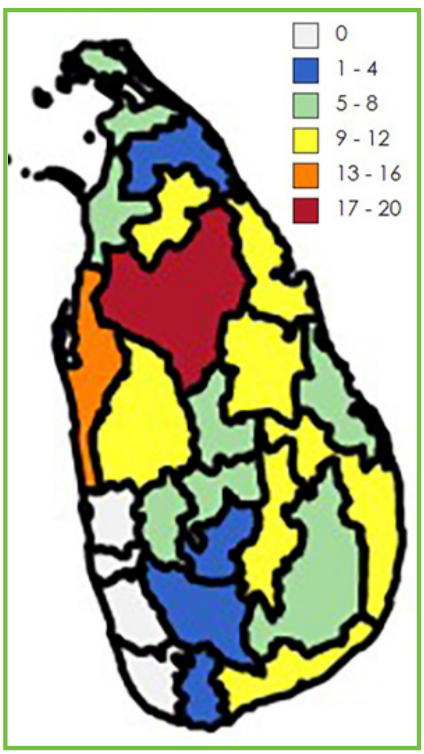


Figure 2: Drought disaster incidence (Source: Zubair *etal.*, Natural Disaster Risks in Sri Lanka: Mapping Hazards and Risk Hotspots, Natural Disaster Hotspots Case Studies, 2011)

Flood warning systems are useful in areas surrounding rivers. During a heavy rainfall and before opening of sluice gates of reservoirs, people living downstream can be informed about floods. For this, sensors can be placed in critical positions along a river to monitor water level. In addition to that, analyses of river basin and relationship between rainfall and stream flow can be used to model the risk of flood, and to issue warnings. A similar type of approach can be applied in early warnings of cyclones estimating changes in pressure and wind speed.

Risk maps

As we know, visual perception is stronger than written and verbal communication. Visualizing risk of a certain disaster according to the geographic divisions, is the primary task of a risk map. Usually, different colours are used to indicate different risk levels, for instance, red for high risk, orange for low risk, green for no risk. The risk index values can be converted to such levels choosing suitable threshold values. The map in Figure 2 shows the incidence of droughts in Sri Lanka for each district. Here, different weights (1.5 - major, 1 - medium, 0.5 - minor) are assigned in aggregating the number of droughts.

These types of risk maps are helpful in managing disaster risks in all its phases. Early preparedness for response and recovery can be planned. Furthermore, long-term policy decisions can also be taken to mitigate possible loss of lives and property damages. For instance, regional authorities can organize awareness programmes on evacuation in tsunami high-risk areas, and more rescue teams can be deployed in flood high-risk areas. Thus, risk indicators and subsequent risk mapping facilitate the decision making process.

In addition to the knowledge of natural disasters, the understanding about risk indicators should also be communicated to school level. It paves the way for learning how to judge a risk and respond accordingly. Further, enthusiasm would be created about the design of risk indicators, in particular, mathematical formulations. Then one can think of an improved version of existing indicators, or new indicators subject to the availability of data. To succeed in this task, reliable data should be gathered and government institutions responsible for disaster management should invest on research and training about risk indicators.

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Understanding the evolution of world's languages and its mathematical modelling aspect

Dr S.A.K.P. De Silva

Language serves as a basic tool for transferring information between individuals and communities. In addition, diverse languages spoken in different societies ingrain different lifestyles, cultural diffusions, traditions, and religions within them. Therefore, languages play an important role in representing different cultures and religions from different parts of the world. There are about 6,500 languages spoken in the world today, where 4% of which are spoken by a majority comprising 96% of the global population. The most popular languages with the highest number of speakers are identified as Mandarin Chinese, English, Spanish, Arabic, etc. Recent studies have revealed that a number of native languages are vanishing at an alarming rate. The 4% comprising the most popular languages continue to attract the rest of the speakers in the near future. Languages such as Gaelic in Ireland, Yupic in Siberia, and Mescalero in the United States account for 25% of the languages that have less than 1000 speakers. Studies predicted that 50% of the languages will become extinct during this century alone. Out of these, the United States has the highest number of native American languages that are threatened to become extinct. Language extinction threats are caused by either language shifts or by the death of the last persons speaking such languages. For example, 85-year-old Edwin Benson in North Dakota, USA died taking with him to the grave the Mandan language.

Edwin Benson, the last speaker of Mandan, from North Dakota

Language Shift

Language shift on the other hand is motivated by socio-economic, educational as well as ideological benefits that are followed by current trends of the world. These benefits spawned the concept of characterizing languages as lower-status and higher-status by societies.

"The drive towards learning a higher status language is not generational, but it is an environmental trait." – Steven Pinker

Steven Pinker, the psychologist, in his book, "How the mind works", explained how a language is clearly an evolutionary adaptation (Figure 1). Evolution of a language influence the evolution of human beings. This explains the possibility of the status of a language being evolutionary and societies getting adapted to higher status languages in lieu of lower status languages.

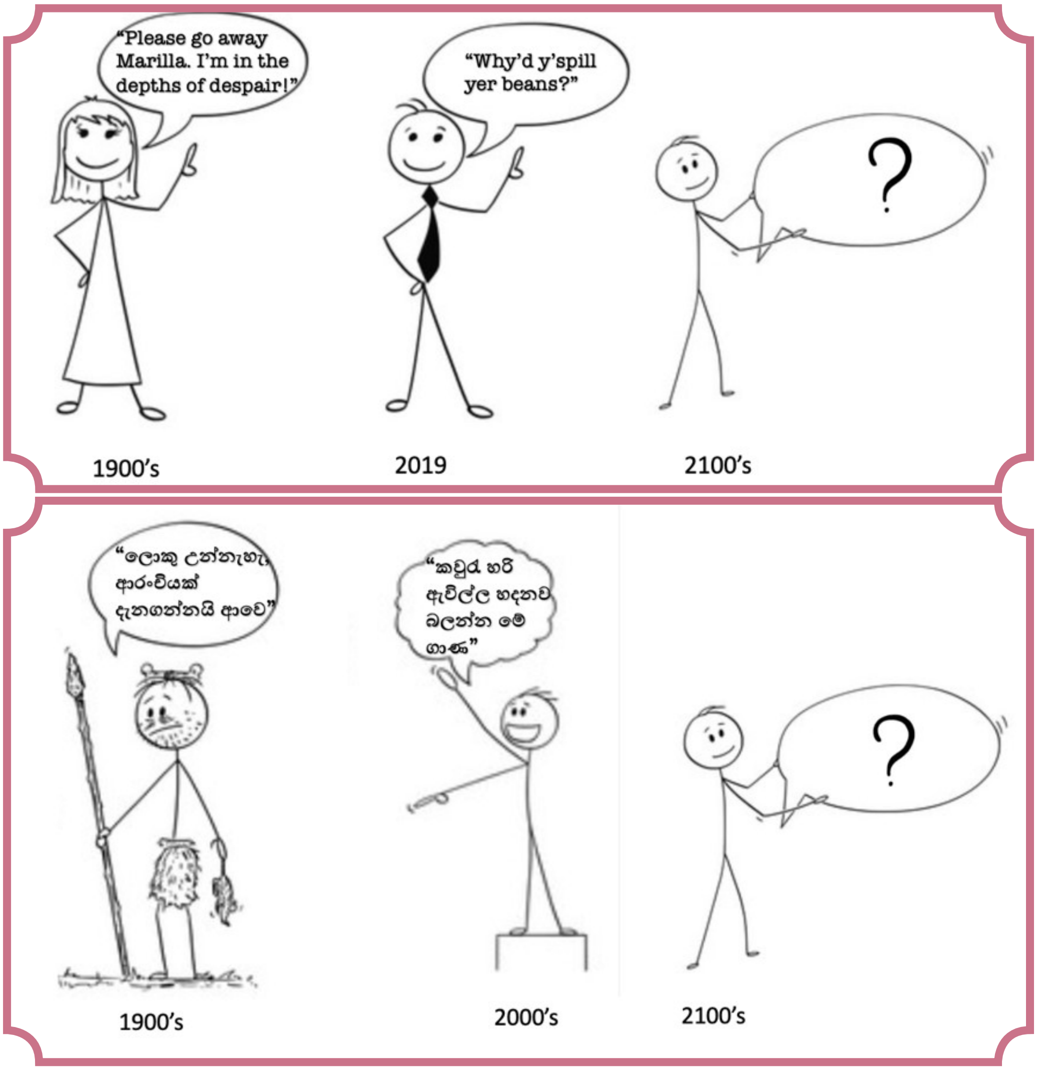


Figure 1: Adaptation of conversational dialects in English (top) and Sinhala (bottom) over time taken from popular plays/movies.

This fact is shown in studies where English language has out-competed Scottish Gaelic, Welsh and Quechua (Figure 2). One of the benefits of such language shifts is the development of the society where everyone has access to a common language. Moreover, English has become the lingua franca, making it an essential tool for education, business trading and technology. Consequently, language shift to English has been intensively studied *via* mathematical models to capture accurately the census data of countries where English has invaded over native languages.

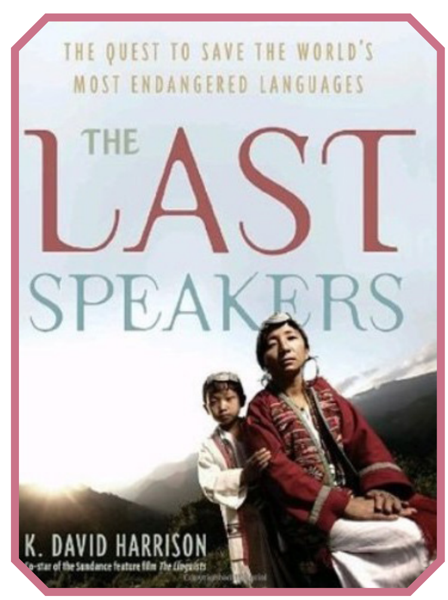
Risk of Extinction

The most popular languages spoken in the world such as English and Mandarin, have successfully attracted most of the population and continue to do so. With the increasing trend of people shifting to these languages, the other endogenous languages are at risk of extinction. However, few languages competing for speakers have put most of the native languages at risk of losing their speakers. Loss of speakers take cultures and religions to the grave. Attention has been drawn to these language extinction risks in recent years. A book based on a television documentary, 'The Last Speakers' by David Harrison, is only one of such pieces of evidence.

"When we lose a language, we lose centuries of human thinking about time, seasons, sea creatures, reindeer, edible flowers, mathematics, landscapes, myths, music, the unknown and the every day".
– David Harrison in "When Languages Die".

Mathematical Modelling

The velocity, the change of rate in position, of an object is found by taking the derivative of position with respect to time. Similarly, the change of rate in population who speak a language, say X, can be modelled by taking



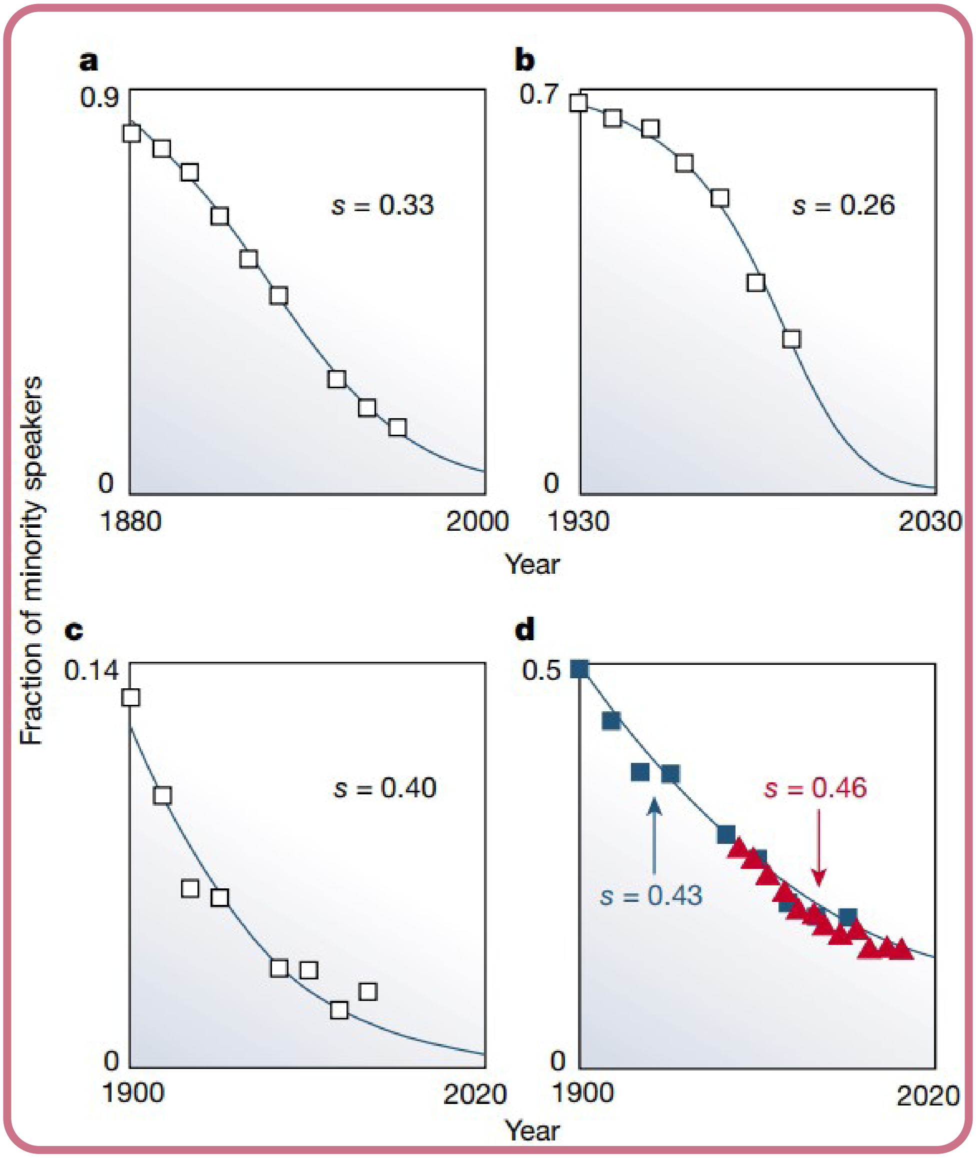


Figure 2: Decline in the fraction of speakers; a. Scottish Gaelic, b. Quechua in Huánuco, c. Welsh in Monmouthshire, d. Welsh in all of Wales

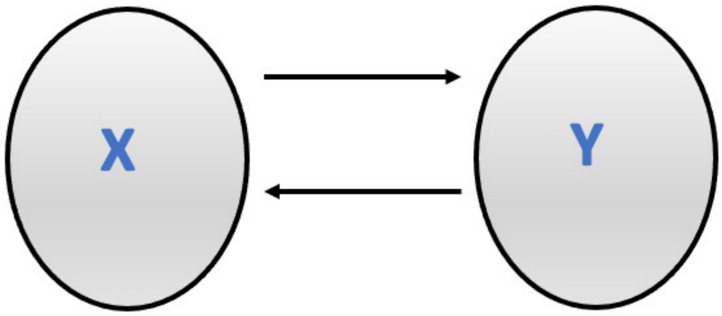
the derivative of population with respect to time. i.e.
dynamics of X=change of rate in language
X=d/dt (population who speak language X)

The approach of mathematical modelling for language shift dynamics, is motivated by looking at languages competing for its speakers, just like species that compete for food. Mathematical models have been developed to formulate language dynamics as a two species (or multiple species) competition, where the higher status language, the stronger competitor, is competing with the lower status language(s), the weaker competitor(s), to acquire speakers. Another approach to model language dynamics is by following the well-known predator-prey models of competition. However, these mathematical models do not account for internal language structures, but only for population fractions. These models have been successful in demonstrating the evolution of language dynamics in monolingual, bilingual and trilingual societies, competing for speakers thus, stressing the risk of extinction of lower status languages *via* numerical simulations performed against real-time census data.

The basic schematic diagram of language shift in a bilingual society is shown in Figure 3, along with its corresponding mathematical equations. In that, X and Y symbolizes the number of populations who speak language *X* and *Y*, respectively. Since two languages *X* and *Y* are at play, we use two differential equations for each language. The population gained for language *X* is equal to the population lost for language *Y* (i.e., $dY/dt=-dX/dt$). The term P_{XY} represents the probability in which the population shift takes place from language X to Y. Numerous mathematical functions can be replaced for these probability terms depending on the nature of parameters at play. This type of model can be used to graphically represent the dynamics of a population who speak a language as given in Figure 1 using census data.

Language Mixing

Another aspect of these language shifts is multiple languages competing to gain speakers, and as a result societies become bilingual and/or multilingual. In other words, people tend to use multiple languages mixed in



$$\frac{dX}{dt} = YP_{YX} - XP_{XY}$$

$$\frac{dY}{dt} = XP_{XY} - YP_{YX}$$

Figure 3: Schematic diagram showing the shift from languages X to Y (left) with its corresponding fundamental mathematical equations (right). The symbol P denotes the probability in which the shift takes place.

conversations. It has been discussed in recent studies how these bilingual and trilingual societies use languages in mixed strategies. It is commonly seen in countries with rich language diversity such as Sri Lanka and India. For example, Sri Lanka has three popular spoken languages - Sinhala, Tamil and English. Thus, it is common to find bilingual speakers who speak either Sinhala and English or Tamil and English and trilingual speakers of Sinhala, Tamil and English. This way both native languages and invaded language start co-existing, resulting in sustaining the native languages. This situation overtime, generates new dialects, resulting in losing certain amount of vocabulary due to lack of usage. This might over time place at risk of losing the sense of those vocabularies, and they will not be carried out to future generations. However, it should be stressed that multilingualism results in less risk of losing speakers of certain languages than speakers completing to shift to a new language. Moreover, recent studies have shown that the ability to speak, think and write multiple languages help in heightening cognitive abilities.

Language Preservation

Due to reasons of endangerment of native languages, it is important to identify language preservation policies by countries and/or societies. One of the ways to preserve languages is documentation. However, translations may cause a loss of information when they become out of use for generations. This may explain the reason behind the development of various forms of religious beliefs through generations. One example is the effort to preserve Buddhism, the religion of the majority of Sri Lankans, through several languages like Pali and Sanskrit, as well as Sinhala in Sri Lanka. It was initially written on *sellipi's*, and it is doubtful if anyone has the ability to read and understand them today. It is observable that there are derivatives of misconceptions and misbeliefs of Buddhism developed through our generations. There are other cases where certain native languages were able to preserve speakers due to ideological benefits influenced through political reasons in a country aided by TV-media and newspapers. For example, one of the latest studies has shown *via* a mathematical model, that in India, Hindi, their native language, is on the rise at a greater rate than English. Other methods of preserving native languages are adding educational programme for younger generations. However, it is a question of whether the younger generation is motivated to follow the educational programme without a valid reason.

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Fuzzy modelling in real-life

Dr (Mrs.) H.O.W. Peiris

Many people consider the invention of electronic computers was the greatest achievement of scientists in the twentieth century, as it has influenced every part of the world and changed the human life in all important areas. The question is, are these computers capable of functioning as the human brain? Even the most sophisticated machines have only a part of the intelligence of the human brain, as they do not show a flexible flow in handling the natural language, and process the vague information.

So why is our brain more advanced and sophisticated than a computer? A general automatic washing machine that is in our homes cannot think. It cannot judge how many clothes to wash? It cannot judge whether the clothes are made up of woven cotton or heavy cotton? It cannot judge whether they are daily washed clean clothes, or dirty or very dirty clothes? After we press the corresponding buttons, it works according to the preset programme like pouring water, adding detergent automatically etc. In our day-to-day life, we use our experience to make the cleaning process more efficient. By simply looking at the clothes to be washed, we have the ability to decide whether this pile of clothes is not dirty, very dirty, dirtier, lighter, not heavy, very heavy etc. According to that, we decide how much of the detergent would be required and whether they could be washed gently or hard scrubbed. Likewise, we find many such vague concepts, so called fuzzy concepts without clear boundaries in daily life. Human brain can easily identify these concepts and make judgements instantaneously whereas computers tend to make precise decisions.

However, we can make computers to understand fuzzy concepts using fuzzy logic. In 1965, Lotfi Zadeh introduced a new concept, the so called fuzzy logic for representing and manipulating fuzziness. This concept facilitates to model our sense of words, our decision making and our common sense. As a result, it is leading to new, more human, intelligent systems.

What is fuzziness?

Let us step into the world of fuzziness with a simple object found in most homes - a bowl of fruits.

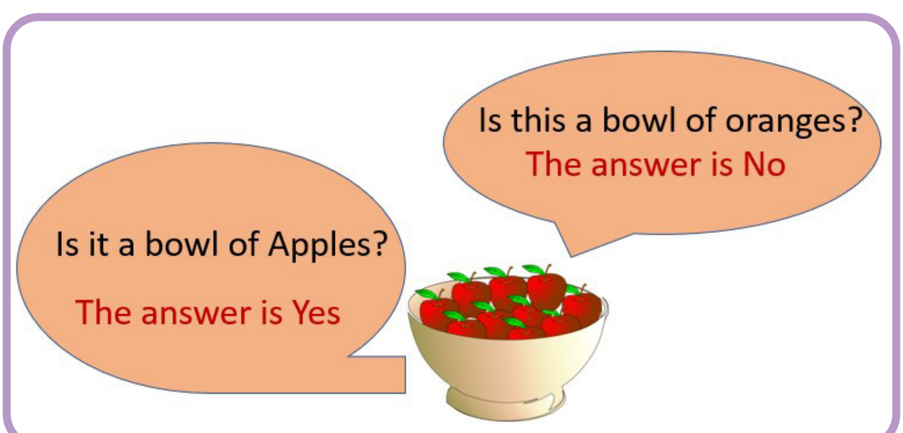

Figure 1: A bowl of apples

According to Figure 1, if I ask

whether this is a bowl of Oranges? Your answer would be 'No'. If I ask is this a bowl of apples? Your answer would be 'Yes'. This is an example of classical logic as we see in conventional computers where the control systems follow the either-or system, or we say as 0s and 1s.

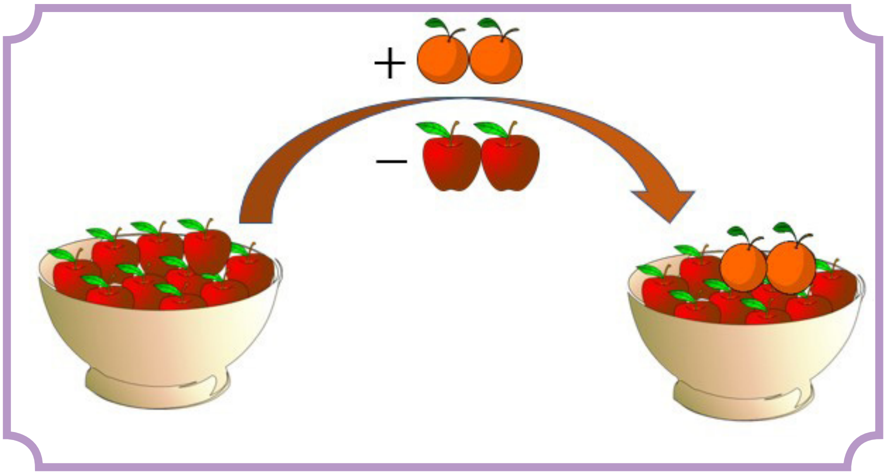

Figure 2: Swapping an orange for one of the apples

This simple environment can be made into a more complex one by swapping an orange for one of apples. If the swapping process continues, we are unable to use the term Yes or No. In fact, we can use fuzzy or vague terms somewhere in between Yes and No. As in Figure 2, the term "slightly" can be used as the answer.

Likewise, once the bowl is totally replaced with oranges, the fuzzy terms like somewhat, sort of, few, mostly and yes/absolutely can be used to represent the answer for the question, "Is it a bowl of oranges?" (Figure 3)

Unlike classical logic, fuzzy logic handles all values between 0 and 1. If you really look at the way we make decisions, even the way we see computers and other machines, it is surprising that fuzziness is not considered the ordinary way of functioning. So how does fuzzy logic work? Fuzzy logic based on the theory of fuzzy sets, set that calibrate vagueness.

Fuzzy sets

The concept of a set is fundamental to mathematics. As to classical set theory, an element must either definitely belong or not to a set. Classical sets run with rigid boundaries. In contrast, fuzzy sets deal with fuzzy boundaries, where each element has its own degree of belongingness to a set. For example, let us consider a cricket team. The collection of players make up the team. Here, the team is the set and players are the elements in the set. The team is lined up in order of height. If you want to find who is tall among the players in the team, as the initial step we set up a certain height as margin. As depicted in Figure 4, the margin is referred as crisp boundary. If a player is taller than the chosen height (margin), he is classified as tall. And if they are not taller than the chosen height, they are classified as not tall. In the context of classical set theory, the set may be defined as "tall" where the elements are players who are taller than the chosen height (margin); "tall' = {player no.4, player no.5}. The players no. 1 to 3 are not included in the set "tall."

According to Figure 4, there is a slight difference between the

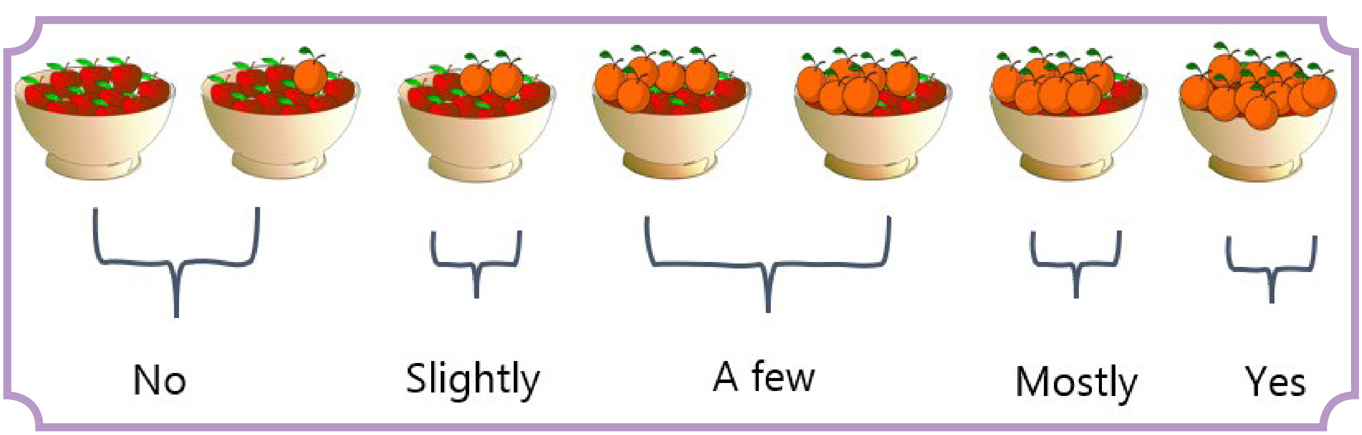

Figure 3: Fuzzy terms between Yes and No

heights of players number 3 and 4. In this case, the vague term "tall" is dealt with a crisp or rigid boundary, which is not always the best way to quantify the information. If fuzzy logic is applied for the set "tall", all the players are included within the set to a certain degree. For example, as depicted in Figure 5, the player no. 2 who does not belong to the classical set (Figure 4)

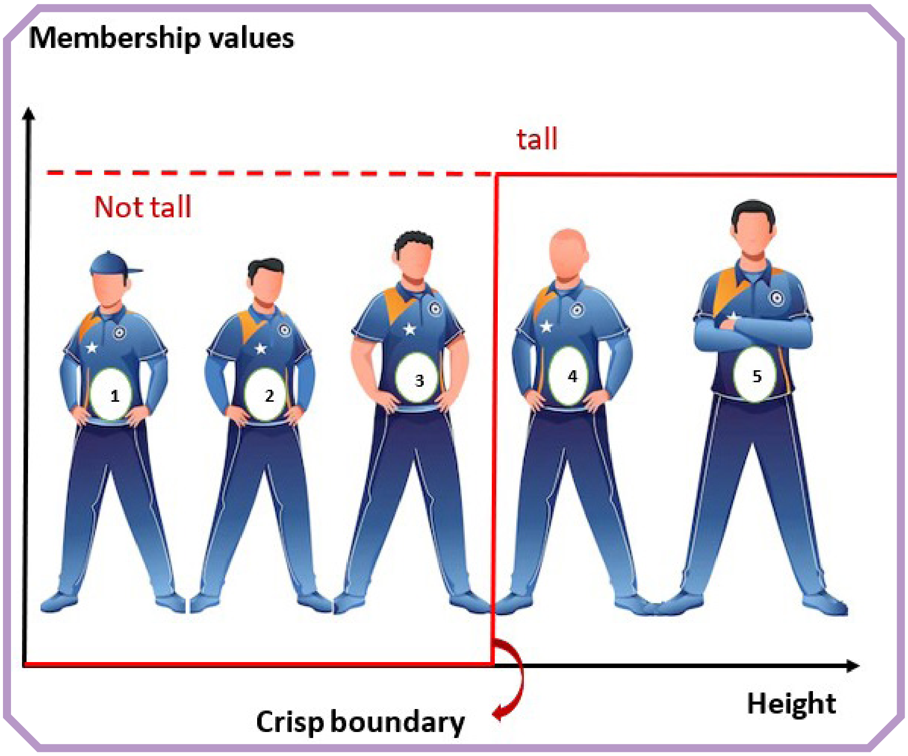

Figure 4: Classical set for "tall"

Fuzzy modelling in real world applications

A mathematical model is simply a theoretical structure that describes a real phenomenon. This system quantifies and manipulate the variables which describe the phenomenon. These models have been widely applied in areas such as science, engineering, finance, clinical work etc. aiming at information analysis, control of systems, evaluation of strategies and decision making. Now the question is what is the role of fuzzy logic in modelling?. In many works, uncertainty is inherent within the variables which lead to make the modelling environment more complex. These variables do not show sharp boundaries, and cannot be handled in classical or statistical methods.

Life insurance scheme

For example, consider a life insurance policy scheme defined for a policy holder. Let us see how the insurance has been defined:

The insurance company offers a nonsmoker bonus of 65% more insurance coverage with no increase of premium if the applicant has not smoked for 12 months prior to application. A bonus of 100% is offered if the applicant:

-- has not smoked for the past 12 months, and
-- has a resting pulse of 72 or below, and
-- has a blood pressure that does not exceed 134/80, and
-- has a total cholesterol reading not exceeding 200, and

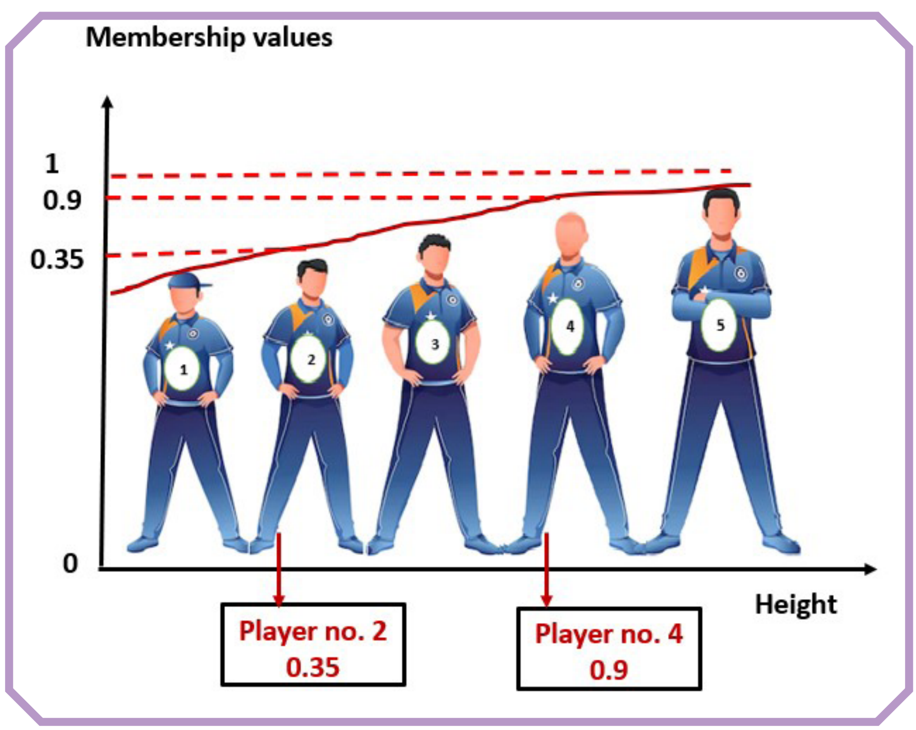

Figure 5: Fuzzy set for "tall"

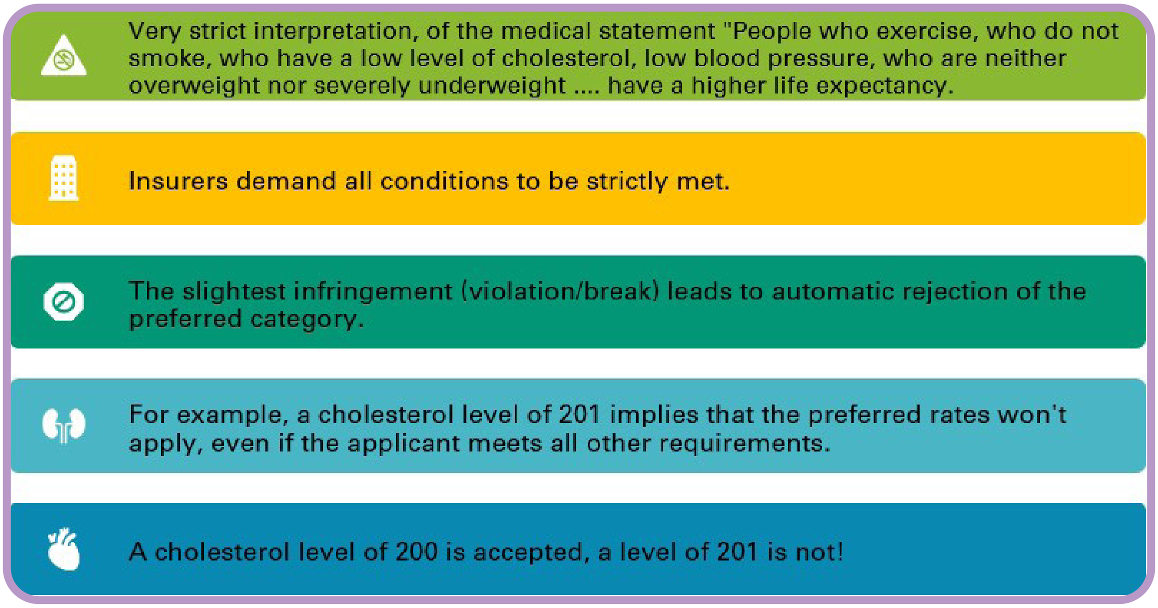

Figure 6: Limitations of insurance policy scheme

-- rigorously follows a 3-times-a-week exercise programme of at least 20 minutes, and
-- is within specified height and weight limits, and
--has no more than one death in the immediate family prior to 60 years of age due to kidney or heart disease, stroke or diabetes.

If you carefully investigate the requirements to be fulfilled by the applicant, the scheme itself presents a very strict medical statement “People who exercise, who do not smoke, who have a low level of cholesterol, low blood pressure, who are neither overweight nor severely underweight .... have a higher life expectancy". According to the limitations given in Figure 6, this insurer seeks all the requirements to be strictly met, for example, the applicant can get rejected if he/ she has a cholesterol level of 201 even if the other requirements are satisfied. A fuzzy set theory can be applied to make the insurance policy more flexible.

For example, in order to reduce the sharpness of boundaries of the cholesterol level, more data should be acquired to understand the behaviour of that variable. The National Institutes of Health nowadays recommends a level of less than 200 mg of cholesterol per deciliter of blood levels, and between 200 and 240 mg/dl are considered to be borderline high. The facts highlighted here are the main characteristics of cholesterol level. Figure 6 depicts how the cholesterol level is quantified using fuzzy techniques, by incorporating the facts given above. You may notice that the representation of cholesterol level is much reliable than that of rigid boundaries. Apart from that, aggregating the effects of these variables/

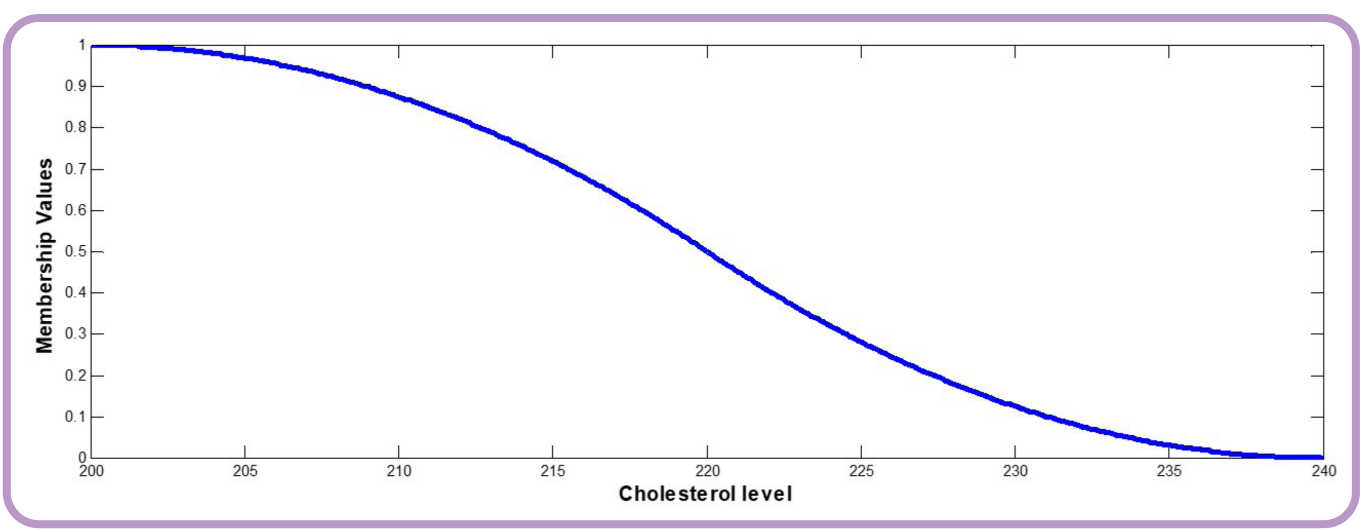

Figure 7: Membership function for cholesterol level

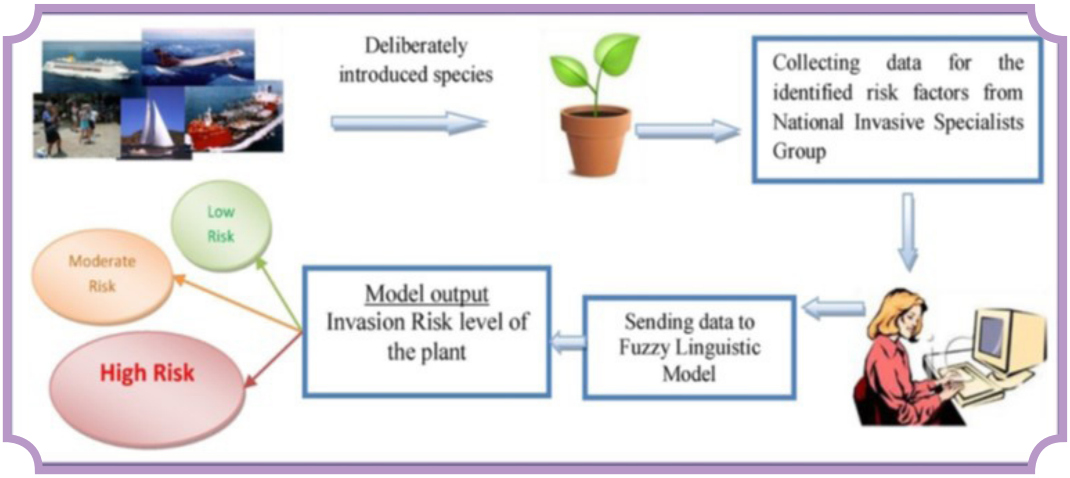

Figure 8: Risk Assessment Model

criteria would lead to make more understandable about the health risk of the applicant. Fuzzy set theory operators like Hamacher, Yager, Dombi etc. may be used to introduce compensation, cumulative effects, and interactions within the variables. For example, the effect of high blood pressure may be amplified (expanded) by the presence of a slight obesity and a cholesterol level mildly above normal, using fuzzy set operators. Let us take another interesting application.

Risk assessment of invasive plant species

Invasive plant species are non-native species, which are deliberately introduced to a new environment, and spread beyond its limits by causing damages to the environment. You may have noticed the plant “common lantana (*gadapana*)” in our neighborhood, and the aquatic plant “water hyacinth *(Japan jabara)*” in many water resources in our country. We have developed a mathematical model to identify the potential of the invasive species using its invasive traits/characteristics. Figure 8 illustrates how the model is executed. Fuzzy set theory is the main technique in developing this model. Due to lack of data, most of the data are in the form of approximations given in linguistics terms. And most of the traits cannot be easily quantified as it is very difficult to handle with conventional mathematical/statistical tools. For example, consider the trait viability of seeds. Assume we have two trees with viability of seeds 6 years and 20 years. In that sense, it is not reliable to put a specific margin to differentiate highly viable seeds or low viable seeds. Both trees have their own impacts towards the invasiveness. Therefore, the treatment is to make these data into a homogenous form, which can be done using fuzzy sets. Fuzzy theory is also capable of handling word uncertainty. In this application, most of the traits are qualitative in nature, and we have designed a model which accepts both qualitative and quantitative parameters and produce the risk level of plant species as the final output.

The applications discussed above are a few instances where fuzzy logic techniques play a vital role in handling uncertainty. This theory is highly applicable for the problems where the uncertainty factor is associated with the impossibility of obtaining sufficient amounts of information with the necessary degree of reliability; the lack of reliable predictions of the characteristics, properties, and behaviour of complex systems that reflect their responses to external and internal actions; the infeasibility of formalizing a number of factors and criteria and the need to take into account qualitative (semantic) information.

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Air pollution modelling

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Air Pollution

Clean air is an elementary prerequisite for all life on this planet. The composition of the present air varies considerably from the chemical composition of the original air, as it existed in pre-industrial era. This means that, at the moment, nowhere on earth is there any original air, which could also be considered clean air. The chemical composition of natural air has shown a gradual variation with existance. The presence of toxic and hazardous particles in the air that can cause harmful effects to living beings and the environment, is called air pollution. There are many factors that contribute to air pollution (Figure 1). Among them, human activities are the main cause for air pollution. Transportation and industrial activities utilize a great amount of energy, and release hazardous pollutants into the atmospheric air. Rapid urbanization, burning of fossil fuels, open burning of garbage waste, and agricultural activities also can make breathing air unsafe. These human activities directly release carbon compounds (CO_2, CO and CH_4), nitrogen compounds (N_2O, NO and NH_3), sulfur compounds (SO_2 and H_2S), and particulate pollutants as the primary pollutants. They also release O_3, NO_2 and HNO_3 as the secondary pollutants. These secondary pollutants are formed in the atmosphere due to the chemical reactions amongst primary pollutants or other constitutes of the atmosphere.

The consequences of air pollution are not immediate. But frequent exposure to polluted air causes respiratory problems, cancers, lung diseases, and cardiovascular problems in human beings. Feeling of tiredness, headaches, sore eyes and frequent occurrence of flu are some of the short-term symptoms of air pollution. These health hazards more often influence children. The cost of medical care and treatment to overcome these health hazards are very high. The poisonous substances in the contaminated air can also cause harmful effects to trees and wildlife. Air pollution is a reason for climate alterations and global warming, and these effects can have destructive effects on the composition of the ecosystems. Acid rain is another negative impact of air pollution which cause an irreplaceable damage to agricultural lands, and to heritage buildings (Figure 2).

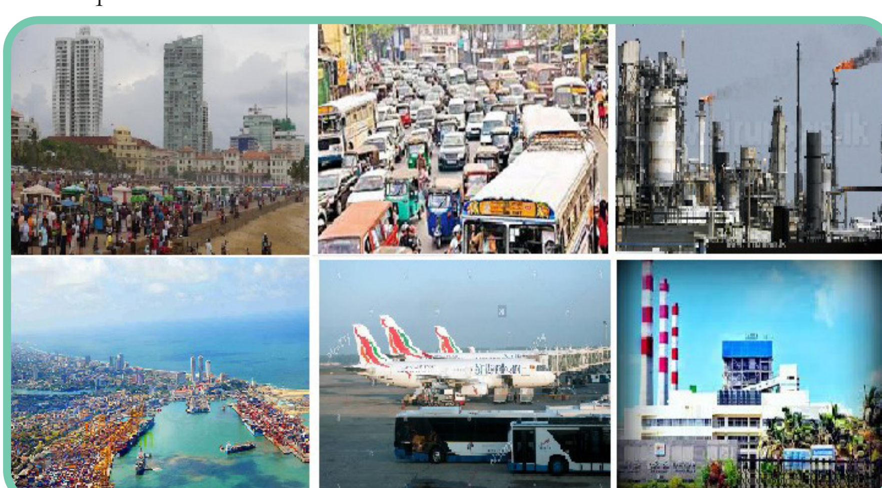

Figure 1: Sources of air pollution

How is Air Pollution Level Measured?

The concentration of aerosols which is also known as Particulate Matter (PM) is an important indicator of long-term air pollution. The pollutant $PM_{2.5}$ are the fine particles that are 2.5μm in diameter and smaller, which are emmited from various sources like power plants, industrial plants, and vehicular emission. The pollutant PM_{10} constitutes the coarse particles that are between 2.5 and 10μm in diameter, which comes from road dust and agricultural operations. $PM_{2.5}$ and PM_{10} pollutants are highly correlated with the health risk of human beings. Therefore, measuring PM concentration levels can help to quantify the air pollution level in our surrounding air.

Usually, air pollution levels are visualized with the aid of air quality indices which can be easily recognized by the public. It works in a similar manner to the speedometer of a vehicle that shows the change of speed from 0 mph to 160 mph. Instead of speed, the air quality index shows us the air pollution level in our surrounding air. Several organizations in the world gather data about the $PM_{2.5}$ and PM_{10} emissions in cities, and convert these records to air quality indices. The air quality indices consist of several categories. A specific colour is assigned to each category. These categories provide the public about the level of cleanliness or unhealthiness of the surrounding air, and recommend actions and health requirements. These air quality index values are broadcast daily through the internet, telephone hot lines, email, and media.

Figure 2: Effects of air pollution

City Planning and Urban Air Quality

Cities are home to most people in the world. According to current records of the United Nations, 54 percent of the world's population live in urban cities, a proportion that is expected to increase to 66 percent by 2050. Urbanization is a process in which cities are designed and subsequently expanded to accommodate the increasing number of people wishing to live and work in urban areas. The major factors affecting urban air quality are the geographical setting, climate factor, human activities, and city planning. New development projects such as new housing schemes, leisure parks and hotels begin to appear in urban areas. In urban planning, urban air quality management play an important role. Thus, it is significant to rank the cities based on levels of air pollution, so that authorities can carry out anti air pollution projects wherever necessary. These projects can include more green spaces and adopt control factors that contribute to air pollution.

Ranking Cities Based on Air Quality

The air quality indices can be used to rank cities and countries according to their air pollution levels. IQAir is an organization which rank the world cities based on $PM_{2.5}$ concentration values. Their air quality indices and health recommendations are shown in Figure 3. According to their statistics published in 2020, the average $PM_{2.5}$ concentration in Sri Lanka was 4.5 times higher than the World Health Organization recommended safe level of 10μg/m^3. Out of 106 countries in the world, in terms of the IQAir, Sri Lanka was ranked to the 30th position in terms of air cleanliness, and categorized as a moderate level country considering particle pollution (Figure 4).

Ranking information is an indicator of quality of living conditions in each place, and therefore, it can be useful for tourists to identify attractive places. However, measuring air quality levels of cities continuously in developing countries like Sri Lanka, is not always possible due to the limited resources available. Therefore, the existing indices are not in an active status for people to be made aware of. A solution to this problem can be provided with the mathematical modelling approach, with the use of indirect factors of air quality such as the number of factors in the area, number of vehicles, population density, etc. However, these indirect measurements are uncertain, because it is not possible to determine the exact levels and values of these measurements. The fuzzy theory provides an approach to deal with arbitrary and uncertain concepts. Due to the uncertain nature of the problem, fuzzy theory is used to solve the problem. This article discusses development of a mathematical model using indirect measurements like the number of vehicles, and the number of factories in the area as an essential task. Such a mathematical model can be used to rank the cities and to identify air quality levels.

US AQI Level		PM2.5 (µg/m³)	Health Recommendation (for 24hr exposure)
Good	0-50	0-12.0	Air quality is satisfactory and poses little or no risk.
Moderate	51-100	12.1-35.4	Sensitive individuals should avoid outdoor activity as they may experience respiratory symptoms.
Unhealthy for Sensitive Groups	101-150	35.5-55.4	General public and sensitive individuals in particular are at risk to experience irritation and respiratory problems.
Unhealthy	151-200	55.5-150.4	Increased likelihood of adverse effects and aggravation to the heart and lungs among general public.
Very Unhealthy	201-300	150.5-250.4	General public will be noticeably affected. Sensitive groups should restrict outdoor activities.
Hazardous	301+	250.5+	General public is at high risk to experience strong irritations and adverse health effects. Everyone should avoid outdoor activities.

Figure 3: IQAir air quality index (Source: *https://www.iqair.com/*)

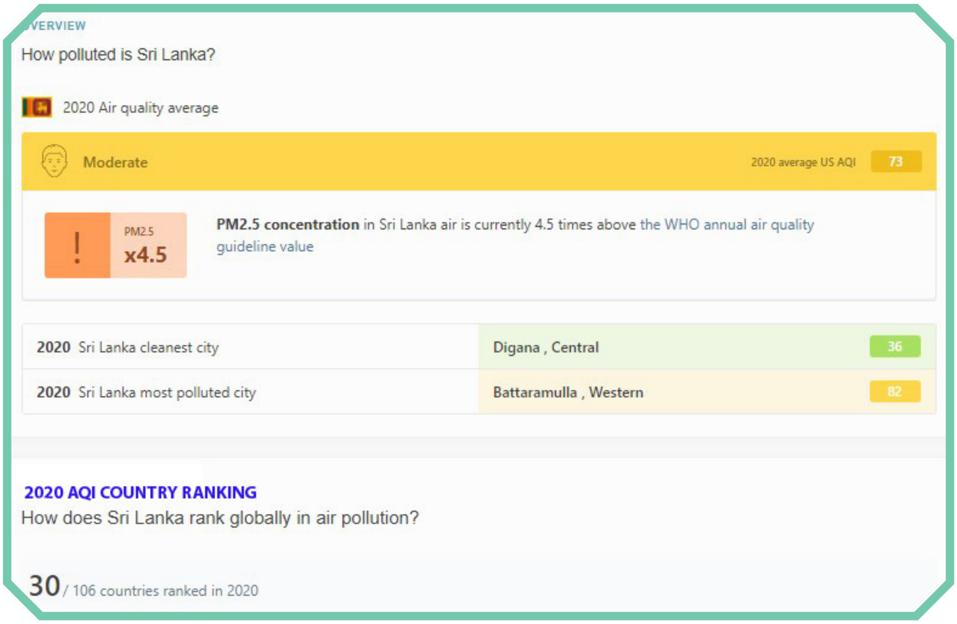

Figure 4: IQAir country statistics (Source: *https://www.iqair.com/*)

Uncertainty and Fuzzy Theory

Let us consider an example to recognize the idea about uncertainty. Who are the tallest boys in Figure 4? How can we define the word 'tallest'? Using this example we can build the idea of uncertainty in the following manner: How do we define

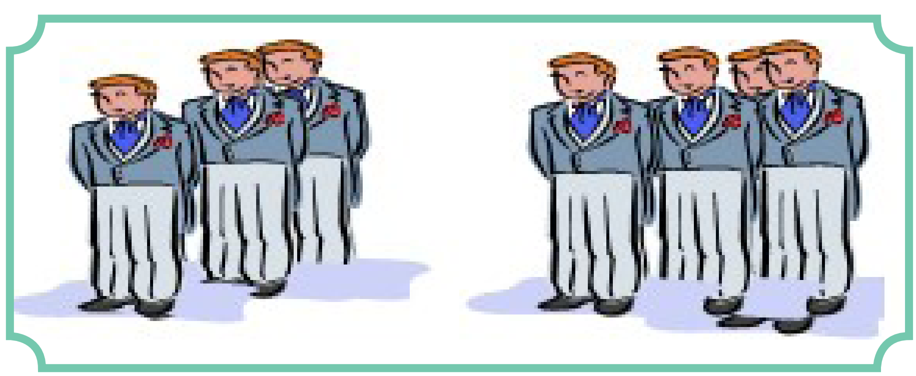

Figure 4: Who are the tallest boys?

somebody is taller than another person. It is by looking at the height. Suddenly if another person joins who is found to be taller than earlier person. Thus, it is difficult to have a concrete criterion to define the tallest.

In conventional mathematical theory, an element belongs either to a set, or it does not belong to a set. The objects require deep understanding of a system, exact models or equations, and precise numeric values. However, most words and evaluations which we use in our daily reasoning are not clearly defined as in the conventional set theory. Therefore, we need knowledge and experience to model unclear and uncertainty ideas in real-world problems. The use of fuzzy theory facilitates us to model these uncertain, vague and fuzzy ideas with the aid of expert knowledge and experience.

As the complexity of the system increases, the ability to make precise and significant statements about the problem behaviour is lessened. Computers cannot satisfactorily solve such problems, as machine intellect still employs Boolean logic. The power of human intelligence results from its capacity to treat fuzzy statements and decisions by adding logical statements and thinking processes. The human brain has numerous intelligent practices and has superior filtering capacity than computers. Therefore, fuzzy theory leads to vagueness in the decision-making processes that are closer to the human intelligent performance.

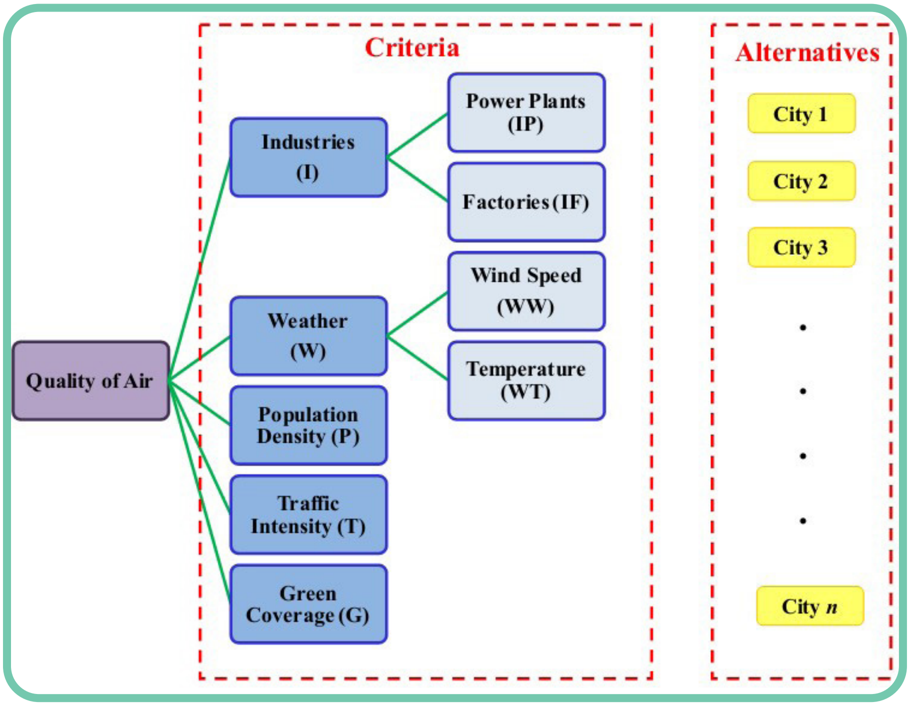

Figure 5: Hierarchy structure of the factors and cities.

Air Pollution Model

Mathematical modelling is a tool that uses mathematical language to describe the behaviour of a real-world problem. Air quality is affected by indirect measurements such as the number of vehicles, number of factories, number of airports, harbours, power plants, population density, weather conditions and available green spaces in the area. Some of these factors have a positive relationship with air quality, while some have negative relationships. Considering these positive and negative relations, five main indirect measurements are selected. They are the number of industries in the area, weather, population density, traffic intensity and green coverage of the area. Some of these factors are further subdivided as shown in Figure 5. To develop the model,

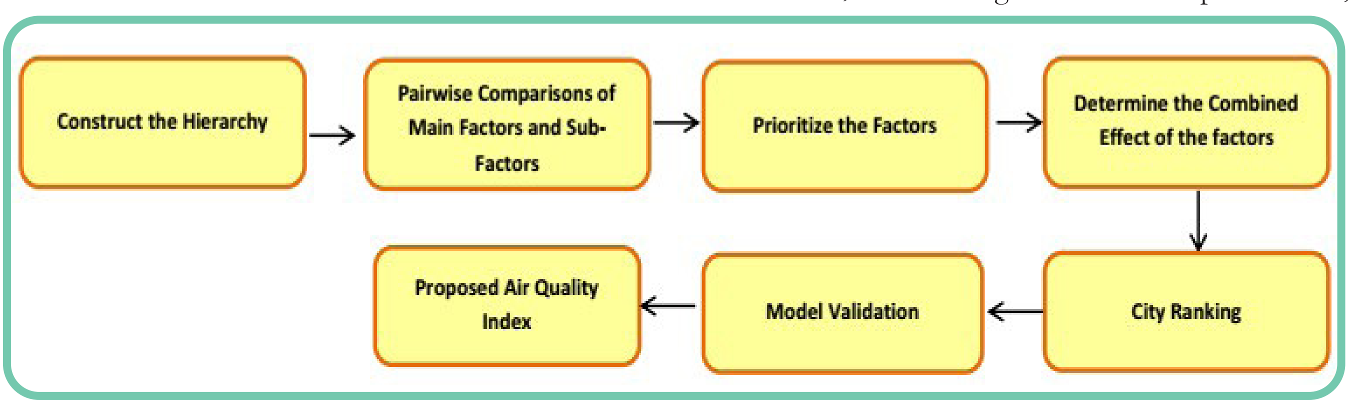

Figure 6: Flow pipe of accessing the air quality level.

Table 1: Proposed air quality index

Air Quality	Fuzzy Value
Good	0.8701 - 1.0000
Moderate	0.7631 - 0.8700
Unhealthy for Sensitive Group	0.7101 - 0.7630
Unhealthy	0.6201 - 0.7100
Very Unhealthy	0.5001 - 0.6200
Hazardous	0.0000 - 0.5000

Table 2: Air quality level of selected cities

City	Air Quality
Tokyo	Moderate
Mexico City	Moderate
New York	Moderate
São Paulo	Moderate
Mumbai	Unhealthy for sensitivity group
Kolkata	Unhealthy for sensitivity group
Shanghai	Unhealthy for sensitivity group
Buenos Aires	Moderate
Delhi	Very unhealthy
Los Angeles	Moderate
Osaka-Kobe	Good
Jakarta	Moderate
Beijing	Unhealthy
Cairo	Unhealthy
Dhaka	Unhealthy
Moscow	Moderate
Karachi	Unhealthy

hierarchical structure as shown in Figure 5 is established with different categories at each level. The goal of this structure is to identify the air quality level in different cities. Therefore, air quality is at the top level of this hierarchy. Factors are positioned at the second level of the hierarchy. Our target is to rank the cities according to the air quality level. Hence, cities are positioned at the bottom level of the hierarchy. The selected cities are Tokyo, Mexico City, New York, São Paulo, Mumbai, Kolkata, Shanghai, Buenos Aires, Delhi, Los Angeles, Osaka-Kobe, Jakarta, Beijing, Cairo, Dhaka, Moscow, Karachi, and Colombo.

Once it has been completed, factors are compared with each other. To do this, we gather expert ideas and experiences. In a pairwise comparison, the decision maker observes two factors by considering one criterion and specifies a preference. These comparisons are made using a preference scale, which assigns numerical values to different levels of preference. The standard preference scales are 1-3, 1-5, and 1-7 point. These scales are associated with linguistic terms. If the importance of one factor with respect to a second is given, then the importance of the second factor with respect to the first is the reciprocal. With the aid of such comparisons, we can prioritize the selected factors.

Figure 6 shows the flow pipe of assessing the air quality level in cities based on indirect factors.

The assessment procedure includes seven sections: (1) Construct the hierarchical structure; (2) Construct the pairwise comparisons between factors; (3) Prioritize the factors; (4) Determine the combined effect of air pollution factors; (5) Determine the rank of selected cities; (6) Validate the model using available data, and (7) Determine the air quality categories.

The proposed air quality index categories are shown in Table 1. Table 2 shows the air quality level in selected cities.

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Mathematical modelling in computing warranty period

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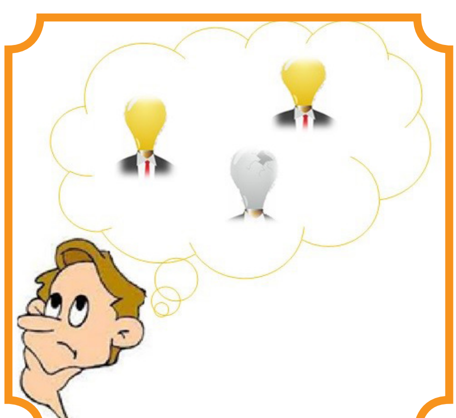
What is a warranty?

When we buy durable products such as washers, refrigerators, air conditioners, computers, and other electronic equipment, office furnishings etc. as consumers we worry about whether this product will fail within very short time, or whether it will survive as we expect. There is the risk of failure of the product within a short period. This is because of the uncertainty of the product failure time. Consumers are not aware of the time of failure when they purchase the product. However, by ascertaining the past data, one can estimate the probable time of failure. So, researchers have studied the past data, and estimated the probable time of failure, and they have modelled the risk of failures. With the use of this risk model, manufacturers provide some assurance to the buyer that the product will adequately meet its performance requirement for a certain period, and that is called a warranty.

In this competitive business world, different brands of the same product have different warranty policies. Thus, the warranty length is an important factor that influences customer satisfaction as well as the demand for the product, as it provides a signal of the quality of the product. A warranty is a contractual agreement between a manufacturer and a customer, that a product or certain of its performance characteristics are free from defects in materials and workmanship. It is a commitment to correct problems, if the product fails during the warranty period. On the other hand, manufacturers can use a warranty to identify and publish the limitations of liability when the product is used in the specific manner. If the product `is not used under the specified normal conditions, the warranty could be annulled, and the manufacturer's obligations are released.



A higher warranty period stimulates the demand for a product. This is because a compressive warranty provides the consumer of a product, the guarantee of a warranty period and higher reliability in quality than a product with a shorter warranty period. Therefore, manufacturers try to offer a variety of product warranties to expand the market share, as consumers consider warranties as one of the most important aspects in making consumer decisions for durable products. On the other hand, manufacturers' concerns are that a more attractive warranty will raise the risk of a product failure causing higher service costs.

Maximizing the profit in the manufacturing business, as the primary objective of the manufacturer, must necessarily mean the consideration of several factors in the process of maximizing profits by offering an attractive warranty: price, warranty length, production volume etc (Figure1). If these factors are

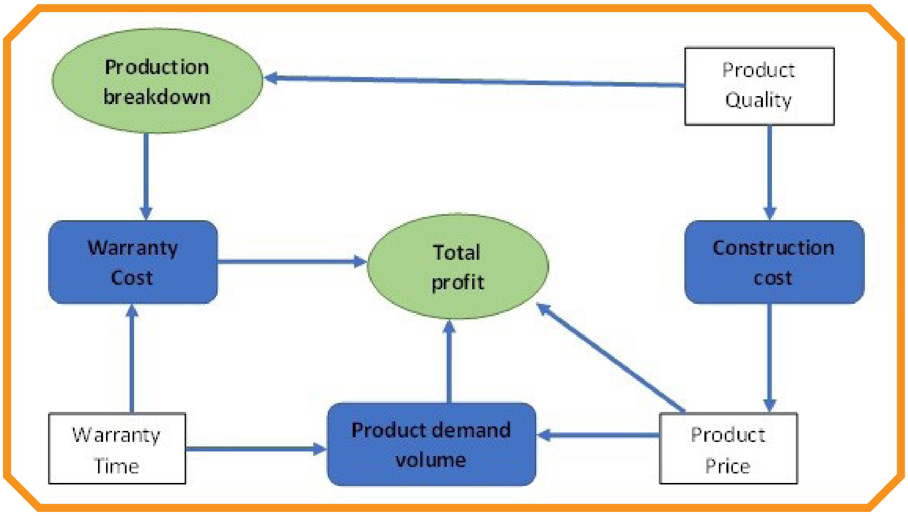


Figure 1: Relationship between different productive factors in total profit

managed in an optimal way, manufacturers can determine the optimal production strategy with the optimal warranty period. For example, if the manufacturer reduces the price and increases the warranty period as a production strategy, then the sales volume and the revenue will increase. However, as a result the warranty cost will also increase. An obvious trade-off between the benefit and the cost of a warranty for a manufacturer is a critical issue for both academic researchers and practitioners.

However, the basis for calculating optimal warranty time is the uncertainty of the time of the failure. As we know, the warranty period starts at the purchasing time and, suppose that the warranty period is *w* years from the purchasing time. Let *T* be the time of the failure from the purchasing time, Figure 2 highlights conditions to exercise the warranty.

Then for a failure that occurs in the warranty period, there will be warranty service cost for the manufacturer, but the manufacturer

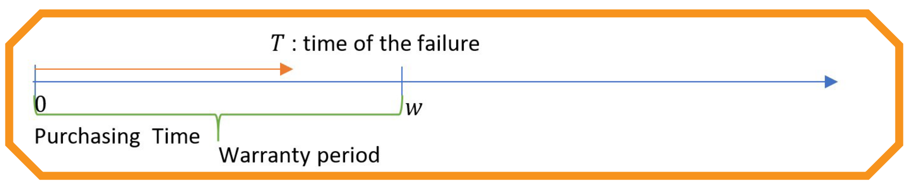


Figure 2: Warranty period

$$\text{Warranty cost to manufaturer} = \left\{ \begin{array}{l} \text{Warranty cost if } T \leq \\ \text{No warranty cost If } T > w \end{array} \right\}$$

does not worry about the failures that occur after the warranty period, as it is not a liability of the manufacturer.

Thus, researchers are studying this randomness of the failure time of the product considering past data, and developing probability models to measure this uncertainty of the failure time (*T*). Such kinds of probability models are referred to as failure time distribution, where the probability of failures occurring over time are described. Four distribution types are supported: Weibull, Normal, LogNormal and Exponentia, for the time of the failure random variable.

In the process of estimating the optimal warranty periods, manufacturers must be prepared to serve for failures that occur during the warranty period, if they are interested about expected number of failures during the warranty period. However, the expected number of failures depend on the failure time distribution. When the manufacture is aware about the expected number of failures in each unit time period of warranty period, then he can estimate the present value of the service cost in advance.

For example let us consider a car battery manufacturer, and suppose that the warranty period is *w* years from the purchasing time. Figure 3 shows the number of failed batteries in each year during the warranty period.

Let C_i: be the unit cost per service capacity in year *i*,

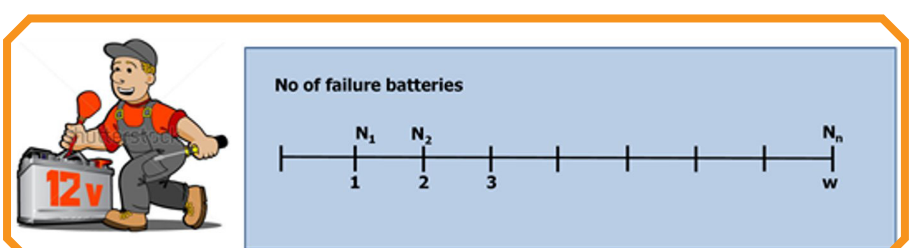


Figure 3: Number of failure batteries

then the expected service cost in year i= $C_i N_i$.

Here the cost of the service is also time dependent since the value of the money is time dependent. The failures will occur at various possibly random points of time in the future, and the cost of the service will also then depend on the time. The mathematical methodology that is used to compare the value of the money at different points of time is theory of interest. As the manufacturer wants to estimate the warranty reserves, before issuing the product, he needs to calculate the value of the total cost of services within the warranty period at the issuing date. Under the theory of interest, the present value of money determines the value of a payment made at a future time in today's money. To find the present value of the cost of service, the discount factor d_n, is used, and assumed that a failure occurs at n years from the initial purchase. Accordingly, as shown in Figure 4, the present value of service cost (C) which occurs at n^{th} year is Cd_n

However, the problem in this process of calculating the present value of the cost of the service, is that n depends on the time at which the failure occured. Basically, duration n is a random variable, because failure time is random.

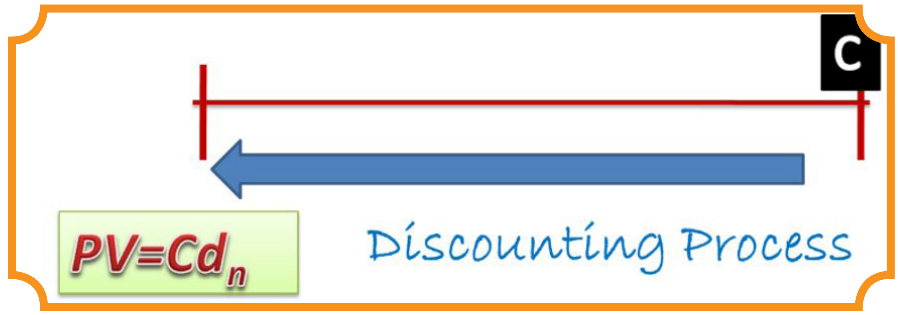


Figure 4: Present value of the cost of service

Figure 5: Random present value of the cost of service

If a failure occurs, then the cost of service will be made, otherwise

$$\text{Present value of Cost} = \begin{cases} Cd_n & \text{If F occurs} \\ 0d_n & \text{If F does not occur} \end{cases} \rightarrow \text{Present value of Cost} = \begin{cases} Cd_n & \Pr(F) \\ 0d_n & 1-\Pr(F) \end{cases}$$

Then we can calculate the expected present value of cost = $\boxed{\underbrace{C}_{\text{COST}}\underbrace{d_n}_{\text{DISCOUNT}}\underbrace{\Pr(F)}_{\text{PROBABILITY}}}$

there is no cost of service to the manufacturer. Thus, as in Figure 5, the present value of the cost of service (C) is equal to Cd_n, only if the failure occurs at n^{th} year.

Then we can use the probability of failure that may occur at n^{th} year to model the randomness of the failure at n^{th} year.

Since within the warranty period there may occur more than one failure, the manufacturer will have to calculate the expected present

$$\text{Expected PV of cost} = C_1 d_1 \Pr(\text{F occurs at 1}) + C_2 d_2 \Pr(\text{F occurs at 2}) + C_3 d_3 \Pr(\text{F occurs at 3}) + \ldots + C_n d_n \Pr(\text{F occurs at n})$$

value of the cost of services for a series of random failures that may occur at different times. Figure 6 describes the series of cost of services for series of random failures.

Then the expected value of the present value of a series of random cost of services is calculated as a summation of expected values of cost of services for each failure.

With this mathematical model, the manufacture is aware in advance about the estimated service cost which is needed in a warranty period.

Why does the Warranty Time Differ from Product to Product?

Generally, the warranty period of CFL blubs vary from three months to one year, while Refrigerators have more than a five-year warranty period. One may be confused as to why the warranty period of different products have different time durations?

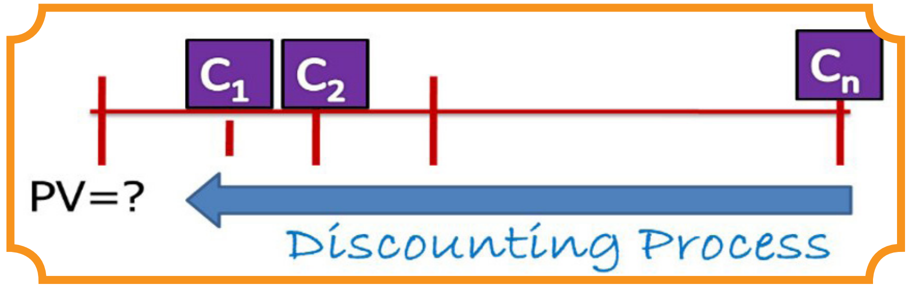


Figure 6: Present value of series of cost

It cannot be assumed that the lifetime of a CFL bulb is same as the lifetime of the Refrigerator. Normally the lifetime of a CFL bulb is limited to 3 months to 5 years depending on its brand, while the lifetime of the refrigerator is limited to 20 years on average. As discussed before, the failure time distribution is the basis for the warranty period calculation. However, the limiting age of lifetime of each product is another factor that decides the warranty period. Nevertheless, there are so many factors which have to be considered in the process of the warranty period calculations.

Types of Warranties

There are several types of warranty agreements available in the market according to the durability of the product type.

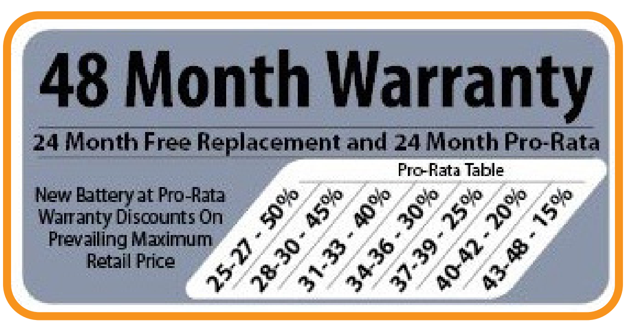


Figure 7: Example of Pro- Rata Warranty (*Source: https://www.exideindustries.com/products/automotive-batteries/warranty-terms.aspx*)

1. Free replacement warranty

With this type of warranty, the manufacturer either replaces, repairs, or reimburses the consumer of the failed product for a time t_w commencing from the time of the initial purchase. This policy is the most common type of warranty for consumer goods, ranging from nonrepairable inexpensive products such as films, to expensive repairable items such as electric items, automobile parts etc.

$$\text{Warranty cost} = \begin{cases} C_T & \text{if } T \leq t_w \\ \text{Not a liability of manufacture} & \text{if } T > t_w \end{cases}$$

Where C_T is the cost of the service such as replacement, repairs, or reimbursements for the failure at time *T*. Here the cost of the service is time dependent, since the value of money is time dependent.

2. Pro-rata warranty

In this warranty, the manufacturer agrees to refund an amount in proportion to the remaining time left for the warranty bounded by tie t_w, commencing from the time of the initial purchase. This policy is appropriate for products such as automotive batteries and tyres, that wear out and must be replaced when they fail.

Figure 7 provides an example of Pro-Rata warranty which is given for a kind of car battery. Here,after the 24 months from the initial purchase, the Pro-Rata warranty is given as follows.

$$\text{Warranty value} = \begin{cases} 0.5V & \text{if } 25 \leq T \leq 27 \\ 0.45V & \text{if } 26 \leq T \leq 30 \\ 0.4V & \text{if } 31 \leq T \leq 33 \\ 0.3V & \text{if } 34 \leq T \leq 36 \\ 0.25V & \text{if } 37 \leq T \leq 39 \\ 0.2V & \text{if } 40 \leq T \leq 42 \\ 0.15V & \text{if } 26 \leq T \leq 30 \end{cases}$$

where V is the value of the battery.

3. Renewing warranty

Upon failure of a product during the warranty period, a replacement is made, and the warranty starts anew. This policy is offered to inexpensive electrical, electronic and mechanical items, where the warranty is contained inside the product's packaging. Figure 8 clearly defines the conditions of a Renewing warranty.

4. Fleet warranty:

A group of *N* items are covered as a lot, for a total period of Nt_w. This type of warranty is applicable to components of industrial and commercial equipment bought in lots as spares, and used one at time until failure.

The above policies can be restricted to a one-dimensional model: here the warranty is described either on the calendar time, or on the usage (measured in another way for

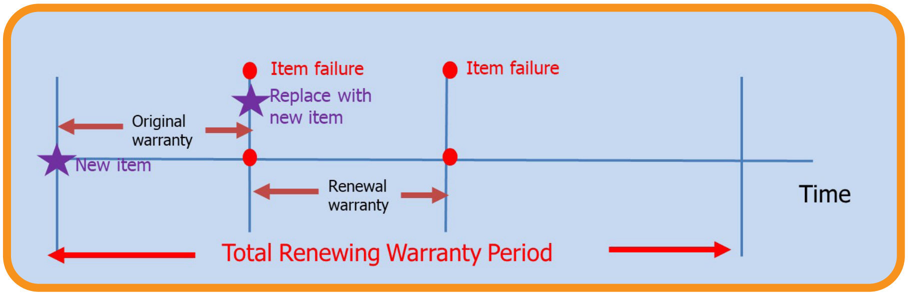


Figure 8: Renewable warranty

example: the distance travelled by automobile) or on a two-dimensional model: the warranty is described in both the calendar time and usage.

The most common two-dimensional warranty policy is the rectangular region policy, where the coverage for maximum time T_w and usage U_w whichever comes first. Consider a rectangular region characterized by $[0,U_w) \times [0,T_w)$, where U represents usage of the product, and T represents time period. We develop a combination warranty policy in 2 dimensions as given by Figure 9.

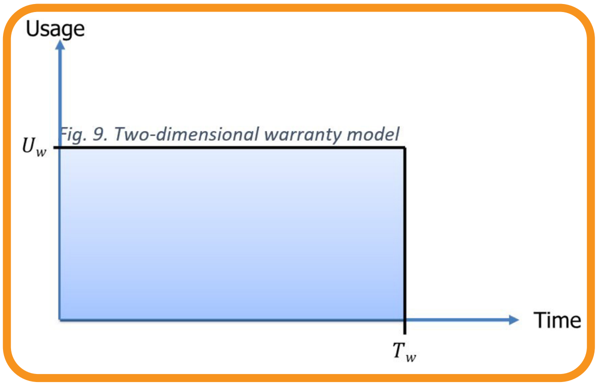


Figure 9: Two-dimensional warranty model

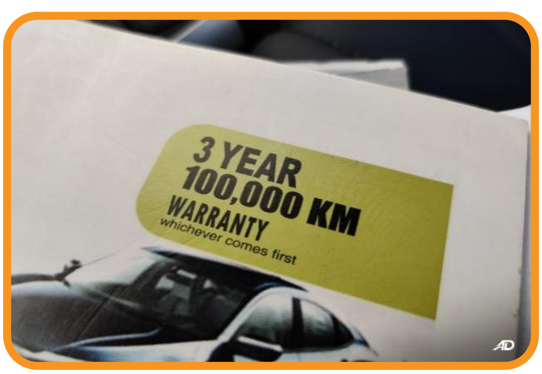


Figure 10: Example of two-dimensional policy

Typical examples of two-dimensional policy appear in Figure 10 which is an automobile warranty defined with two-attributes say, 3 years and 100000 km. Another example is in industrial machinery in which the two attributes may be the number of products produced and the age of the machine. 2-D warranty is valid till either of the attributes or both are exceeded. However, in some industries they offer one dimensional warranty policies having limited range of usage. Heavier users, such as commercial users, generally are offered a shorter warranty time whereas the personal users are offered a longer warranty time.

However, the mathematical modelling process is more complex in two-dimensional warranty policies than in one dimensional warranty policy, as the former contains more variables. In the process of the calculation of the optimal warranty length, one should consider the dimensions of the warranty policy as well as the primary objective of the manufacturer: eg. maximizing profit and consumer satisfaction.

As the warranty is an agreement between the consumer and the manufacturer; the two parties have different objectives: the consumer interest is to have a lengthy warranty, however, the manufacturer wishes to maximize his profit. Therefore, to determine the optimal warranty period, one should consider the manufacturers' goal as well as the customers' satisfaction, and the characteristics of the warranty policy. However, here it is mainly focused on the basis for warranty period calculation: that is, failure time distribution, which is related to the mathematical concept called survival analysis and modelling.

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Mathematics of life and death: COVID-19 pandemic

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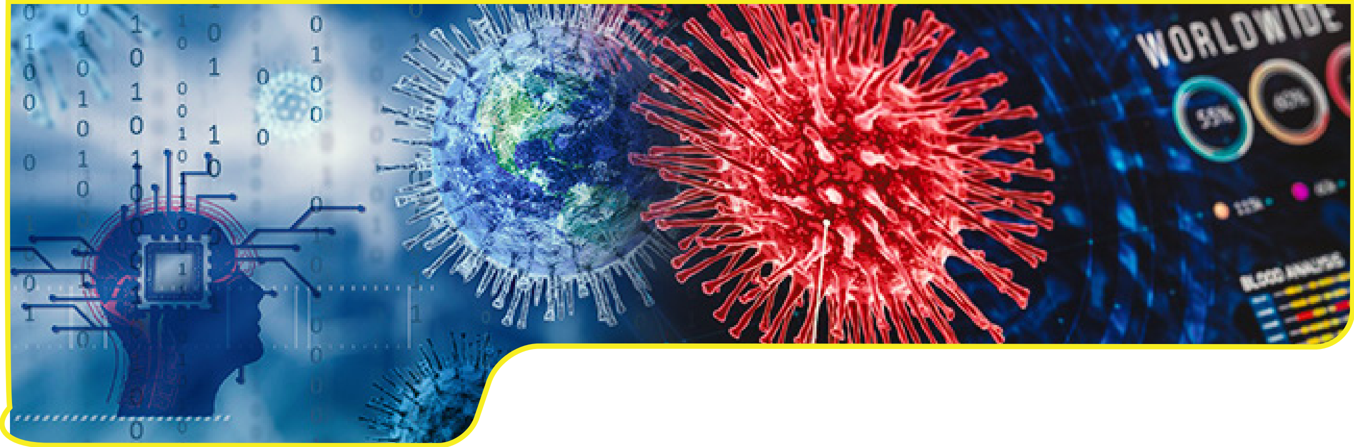

Impact of COVID-19

As we all know, COVID-19 pandemic (also known as coronavirus pandemic), is a preceding global pandemic of coronavirus disease 2019 (COVID-19), caused by the acute respiratory syndrome coronavirus 2 (SARS-CoV-2). It was first identified as an outbreak in the Province of Wuhan, China in December 2019. The World Health Organization (WHO) officially declared COVID-19 as a pandemic in March 2020. This deadly communicable pandemic has affected our lives, economy as well as the entire world in many ways. The impact on the economy and social status of human beings by this pandemic was very destructive. Many people were at a risk of being extremely poor due to various consequences of this disease. To control the transmission of the virus, governments in many countries, regions and localities have implemented strict policies such as: social distancing, work from home, travel restrictions, lock-down or curfew, wearing masks, etc. These policies, in conjunction with voluntary behavioural changes, have resulted in unusual changes in our day-to-day activities, such as travel restrictions, the closing down of business activities and a considerable increase of time to be spent at home.

Importance of Mathematical Modelling in COVID-19

Have you ever thought about why mathematical modelling is so important in pandemics such as COVID-19? Though the virus originated in China, it is common knowledge that it was transmitted to Sri Lanka by travelers from risk areas. How did the disease spread in the country? Why could not we stop or control the transmission and spread of the disease in Sri Lanka? What factors enhanced disease spread? Probably now you already know why we are wearing masks. But are you aware when to stop wearing masks? What will happen if we stop wearing masks now? What will happen if I wear a mask and if half of my classmates do not wear a mask? How long are we going to stay in a lock-down situation? How do government officials decide when to impose lock-downs, and when to reopen? Why are they sometimes imposing lockdowns or curfews only to some selected provinces?

Have you ever thought of the answers to these questions before? Actually, Mathematics can help us to answer these questions. In the early stages of an outbreak of an infectious disease, specifically as in the case of a novel virus like COVID-19, there is a considerable amount of uncertainty about the virulence of the virus, pace of the spread, and the effectiveness of the control measures. By using mathematical modelling based on the number of airline passengers, and the number of infected people reported outside China, provided early estimates about the size of the outbreak of COVID 19, that helped WHO to take immediate action and announce COVID-19 as a pandemic situation.

Nevertheless, simulation results of such models aided policy makers to make some key policy decisions to take control of the spread. Further, some of the models were used to spot the high-risk countries, and to take necessary measures accordingly by analyzing country wise data. Further, now there are many attractive interactive tools designed, based on mathematical modelling, available to help authorities to recognise patterns of the disease spread, and even to effect border screening, to effectively detect imported cases. Sections below will discuss the most important uses of mathematics in disease spread.

Reproduction Number

What if only one person is infected with COVID 19 in a population, and how do we quantify how many other people will be infected? For this purpose, there is a quantity called basic reproduction number (R_0) in epidemiology, which indicates the average number of susceptible (those people who are now not infected but are at a risk of getting infected), to whom a single infected person is able to spread the disease. The quantity

Table 1 : Data of the number of COVID 19 infected individuals

Date	Number of infected individuals
01-01-2022	587,245
02-01-2022	587,596
03-01-2022	587,935
04-01-2022	588,300
05-01-2022	588,929
06-01-2022	589,479
07-01-2022	590,063
08-01-2022	590,651
09-01-2022	591,231
10-01-2022	591,667

R_0 indicates the average number of people to whom a single infected person spreads a disease in a population of susceptible people. For example, suppose that a contagious disease is spreading in the city of Colombo. Before the identification of disease spread, everyone in Colombo was susceptible, and an infected person had arrived in Colombo from another country, and initiated the disease spread to the locals. If the value of R_0 is computed to be 2 and the time that a person is infectious on one day (and then they recover), that person on an average, will spread the disease to two other people, before recovery. These newly infected people will also spread the disease at an average of two or more people each before recovering, and this will continue in tree-like structure. In this simple situation we can easily estimate how many infected people there are during a certain period. And when the reproduction number is greater than 1 (R_0>1), the number of infected individuals grow exponentially. If we take the same value as before to R_0, R_0=2, and an infectious period to be only one day on the first day, there will be only one infected person who imported the disease, and by day 2 there will be two other infected persons. Note that in this case infected people recovered in one day. By day three, there will be 4 infected, and by day 4 there will be 8 infected. These numbers are rising only from one initial infected case, and there will be many people infected, so that by extending the pattern we can determine how bad the situation becomes (Figure 1).

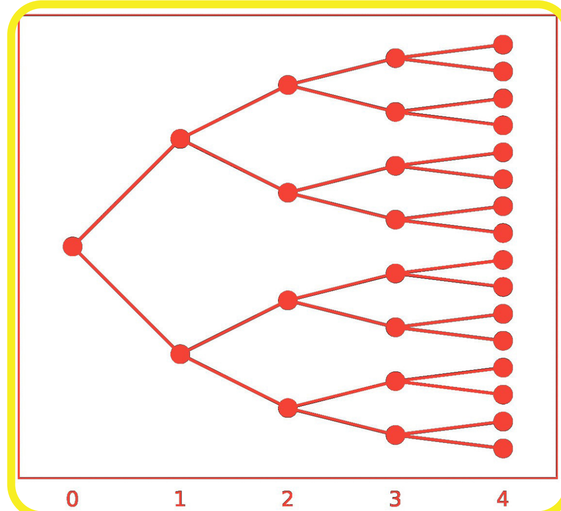

Figure 1: The number of new infections generated when the basic reproduction number is 2. Cases of disease are represented as dark circles.

In a reverse case where R_0<1 represents the case of one infected person infecting less than one person per day on an average, there will be few or no infections each day, and hence the disease will cease to exist over time. How long this continues, and whether it happens, depend on the size of a population, and the in-person contacts of the people in the population. The reproduction number is not a biological constant for a particular disease, and also it is affected by other factors such as environmental conditions, behaviour of the infected population, duration of infectivity of affected people, the number of susceptible people in the population that the infected people contacted etc.. This value is usually estimated *via* mathematical models, and these estimated values are highly dependent on the model used and the values of the other parameters that are used in the model.

4. Simple Mathematical Model

Suppose that we need to construct a simple mathematical model to determine the number of infected people in a certain population (for example in Sri Lanka) after a certain time period. Table 1 shows the data of the number of COVID 19 infected individuals in Sri Lanka taken from the situation reports

Table 2: Rate of change of number of reported COVID 19 infected individuals.

Date	Rate of change of Number of Infected individuals
02-01-2022	(587,596-587,245)=351
03-01-2022	(587,935-587,596)=339
04-01-2022	(588,300-587,935)=365
05-01-2022	(588,929-588,300)=629
06-01-2022	(589,479-588,929)=550
07-01-2022	(590,063-589,479)=584
08-01-2022	(590,651-590,063)=588
09-01-2022	(591,231-590,651)=580
10-01-2022	(591,667-591,231)=436

published by the epidemiology unit of Ministry of Health Sri Lanka. The rate of change of the reported COVID-19 cases are computed as Formula 1.

In this example the rate is computed per day. Therefore Formula 2 and 3.

The above Formula 3 can be expressed mathematically as, $\Delta I/\Delta t=(I_2-I_1)/t_2-t_1$, where ΔI represents the change in infected individuals and Δt change in time.

Table 2 shows the rate of change of number of reported COVID 19 infected individuals for the data represented in Table 1.

The graphical representation of the rate of change of the infected individuals for the given data is shown in Figure 3. The red colour line shows the linear trend line of the rates. Hence, we can deduce that the rates are increasing over time and hence the disease will continue spreading.

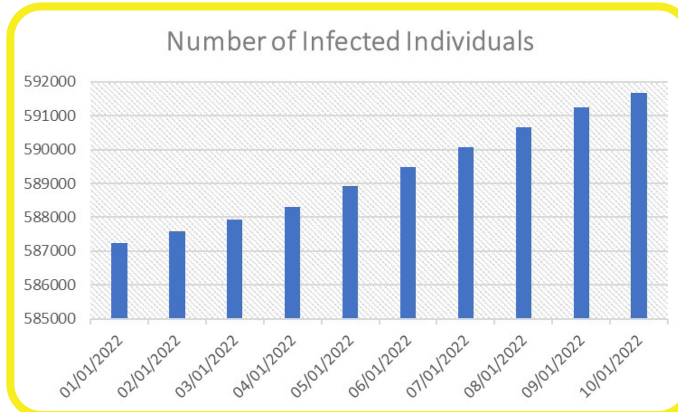

Figure 2: Number of infected individuals from 01-01-2022 to 10-01-2022.

If we assume that the rate of change of infected individuals is decreasing then there must be a decreasing trendline. To show that scenario, a table is generated as follows.

When the rates are compared a decreasing trend can be seen. Figure 2 shows the decreasing scenario, and the linear trendline shows that the disease is decreasing slowly. Hence, we can expect a reduction of the infected individuals over time.

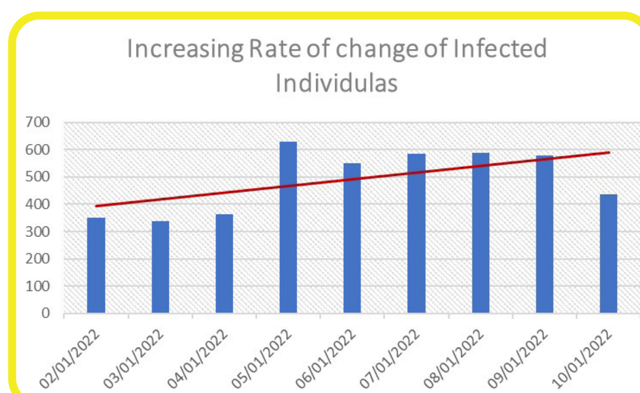

Figure 3: Rate of change of the infected individuals for the given data as in Table 2.

$$\textit{Rate of change of Infected} = \frac{\text{(Number of Infected People in Day 1-Number of Infected People in Day 2)}}{\text{Difference of Days}} \quad ....(1)$$

Formula 1

$$\textit{Rate of change of Infected} = \frac{\text{(Number of Infected People in Day 1-Number of Infected People in Day 2)}}{1} \quad ....(2)$$

Formula 2

$$\textit{Rate of change of Infected} = \text{(Number of Infected People in Day 1-Number of Infected People in Day 2)} \quad ....(3)$$

Formula 3

Table 3 : Rate of change of number of reported COVID 19 infected individuals

Date	Rate of change of Number of Infected individuals
02-01-2022	351
03-01-2022	345
04-01-2022	330
05-01-2022	300
06-01-2022	295
07-01-2022	280
08-01-2022	272
09-01-2022	261
10-01-2022	258

By using the rate as calculated above, we can model a simple equation to compute the number of infected individuals in a population.

Let the number of infected individuals at time t be *I(t)*, and the individuals at time t+Δt be I(t+Δt). At any time *t*, the rate at which the infected population changes (dI(t))/dt ((dI(t))/dt=(lim)┬(Δt→0) (I(t+Δt)-I(t))/(t+Δt-t)=(lim)┬(Δt→0) (I(t+Δt)-I(t))/Δt) is proportional to the number of infected individuals at time *t*. If the number of infected individuals are higher, then the rate is also higher and if the number of infected individuals is less, the rate is also low.

This is a very simple model that shows how the infected numbers are changing with the rate. However, in the real situation, there are many other factors to be considered. This simple model must be incorporated with many other factors to match the real situation, and hence we get a more sophisticated equation or a system of differential equations. By using computer programs this can be simulated and hence, numerical results can be obtained. These results can help us to understand how the disease dynamics happen, and they will help policy makers and decision makers to make decisions such that it helps to reduce or control the disease spread. Figure 4 shows the computer simulated numerical simulation for such a system of differential equations derived *via* compartmental modelling. The blue, green and red dots illustrate the dynamics of susceptible, infected, and recovered populations over time. Mathematics play an important underlying role in obtaining the model and the simulations. Already, there are many softwares created using the mathematical concepts to model COVID-19, and the users need only to feed the programme with required parameters to obtain the numerical results as in Figure 4.

$$\frac{dI(t)}{dt} \propto I(t)$$

dI(t))/dt =k I(t), where k is the proportionality constant

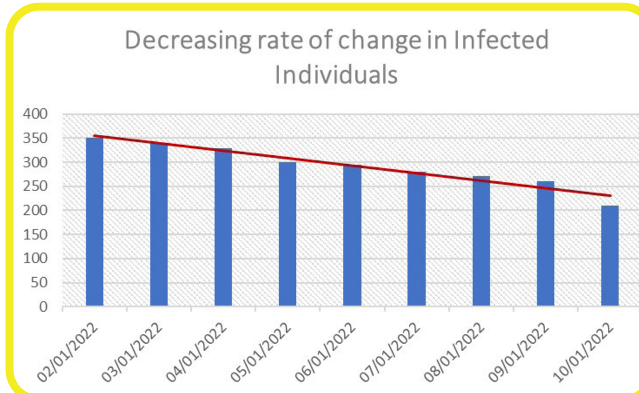

Figure 4: Rate of change of the infected individuals for the given data as in Table 3.

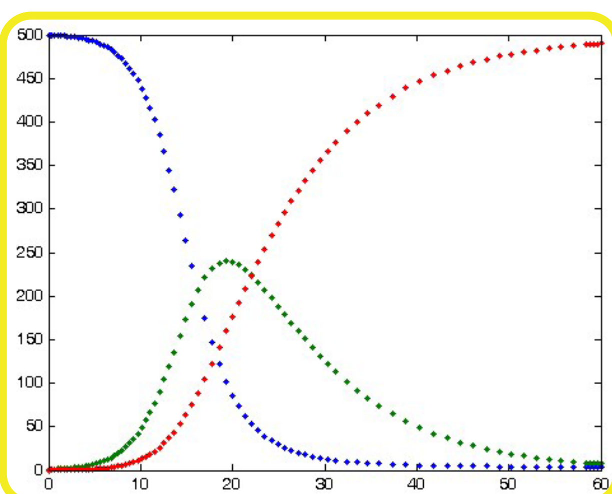

Figure 4: Simulation results of SIR- compartmental model.

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What have you learnt from the Vidurava 2022 January - March Q_1 Issue? Scan your own memory!

1] Mathematical Models (Modelling)

True or False?

1.Mathematical Modelling is the art of translating real world problems in mathematical terems usually in the form of equations to make Predictions.

2.A mathematical world consists of similar mathematical objects such as systems of equations,or a set of differential equations.

3.Principles of mathematical modelling and the steps to be taken are summarized by raising very simple questions and answering and answering such questions.

4.Coupling mathematical modelling with ecological dynamics provides a poor fisheries systems.

5.Mathematical modelling can be considered as a multidisciplinary action tool to handle reak problems.

2] Mathematics for Quantifying Disaster Risk

True or False?

1.Disaster management can be identified as the process in minimizing damages subject to available facilities.

2.The main utility value of a risk indicator is to assess the potential damages.

3.In an additive structure we do not need all the sub-indices to be in the same units, where it is necessary in a multiplicative case.

4.Usually earthquakes with Richter scale values less than 6, lead to structural damages and collapse of buildings.

5.Visualizing the risk of a certain disaster according to the geographic divisions is the primary task of a risk map

3] "Understanding the Evolution of World"s Languages and its Mathenatical Modelling Aspect

True or False?

1.There are about 6500 languages spoken in the world today where 4 % of which are spoken by a majority comprising 96% of the global population.

2.Language extinction threats are caused by either language shifts or by the death of the last person speaking such languages.

3.The drive towards learning a higher status language is generational, but it is not an environmental trait.

4.Most popular languages spoken in the world such as English and Mandarin have successfully attracted most of the population and continues to do so.

5.An approach to model language dynamics is by following the less known predator – prey models.

4] Fuzzy Modelling in Real Life

True or False?

1.We cannot make computers to understand fuzzy concepts using fuzzy logic.

2.Unlike classical logic, fuzzy logic handles all values between 0 and 1.

3.The concept of a set is fundamental to mathematics.

4.A mathematical model is simply a theoretical structure that describes a real phenomenon.

5.Invasive plant species are native species which are not deliberately introduced to a new environment.

5] Air Pollution Modelling

True or False?

1.Frequent exposure to polluted air causes respiratory problems, cancers, lung diseases and cardio-vascular problems.

2.The concentration of aerosols which are also known as particulate matter, are not important indicators of long term air pollution.

3.Air pollution levels are visualized with the aid of quality indices which can be easily recognized.
4.Due to that uncertain nature of the problem of air pollution, the fuzzy theory cannot be used to solve problems.
5.Mathematical modelling is a tool that uses mathematical language to describe the behaviour of a real world problem.

6] Mathematical Modelling in Computing Warranty Period

True or False?

1.Consumers are aware about the time of failure when they purchase the product.
2.A higher warranty period stimulates the demand for a product.
3.In the process of estimating the optimal warranty periods, manufacturers must be prepared to serve for failures that occur during the warranty period.
4.Under the theory of interest, the present value of money determines the value of a payment done at a future time in today's money.
5.The limiting age of a lifetime of each product is not the factor that decides the warranty period.

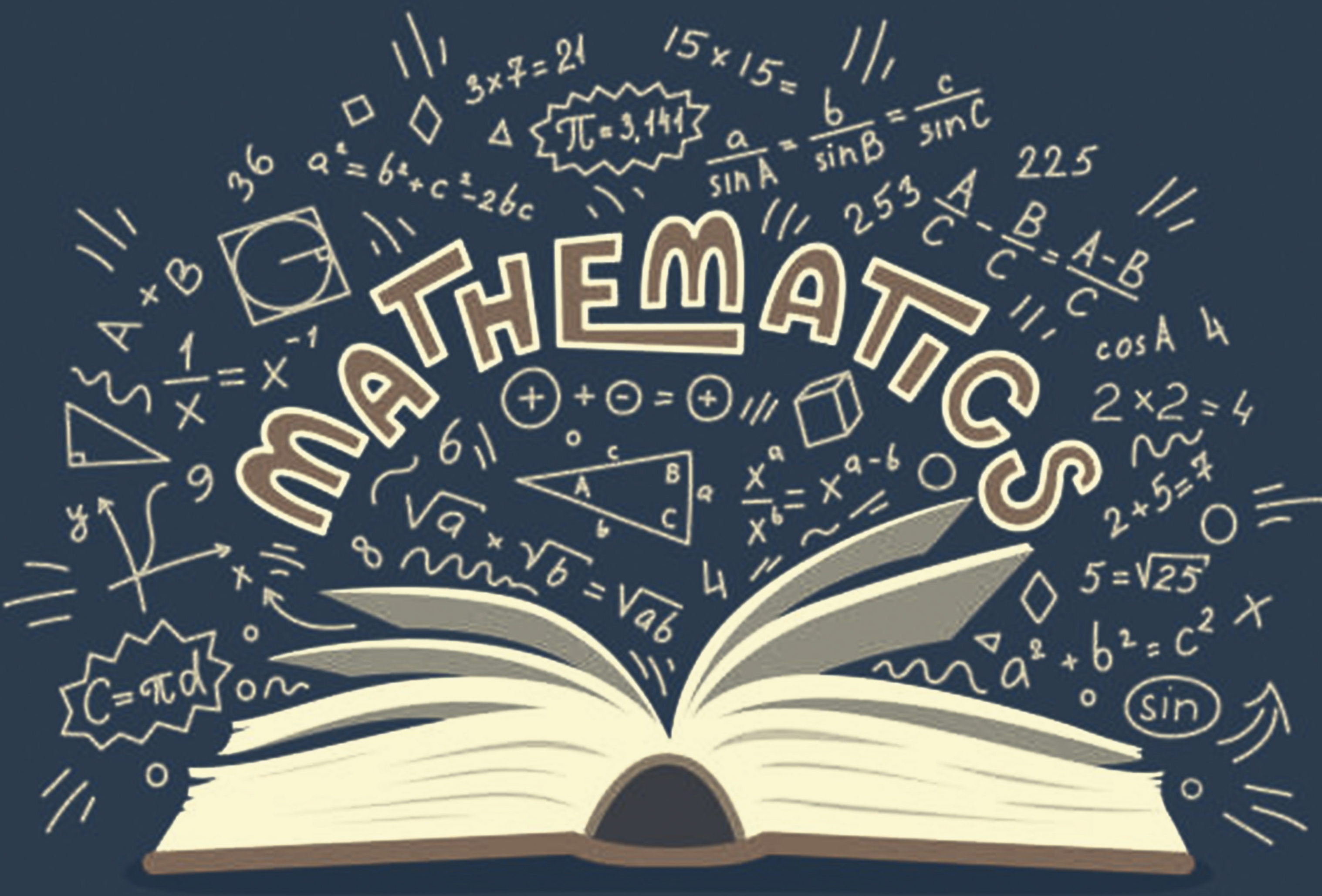
7] Mathematics of Life and Death: Covid – 19 Pandemic

True or False?

1.Many people are at a risk of being extremely poor due to consequences of this disease.
2.By using mathematical modelling based on the number of airline passenger, and the number of infected people outside China, provided early estimates about the size of the outbreak.
3.The reproduction number is a biological constant for a particular disease, and is not affected by other factors.
4.By using computer programmes, a system of differential equations can be simulated, and hence, numerical results can be obtained.
5.State policies in conjunction with involuntary behaviour changes, have resulted in popular changes in our day to day activities.

Answers

01)1. True,	2. True,	3. False,	4. False,	5. True
02)1. True,	2. True,	3. False,	4. False,	5. True
03)1. True,	2. True,	3. False,	4. True,	5. False
04)1. False,	2. True,	3. True,	4. True,	5. False
05)1. True,	2. False,	3. True,	4. False,	5. True
06)1. False,	2. True,	3. True,	4. True,	5. False
07)1. True,	2. True,	3. False,	4. True,	5. False



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